Chapter 3 Polynomial Functions

Section 3.1 Characteristics of Polynomial Functions

Section 3.1 Page 114 Question 1

A polynomial function has the form
$f(x) = a_nx^n + a_{n-1}x^{n-1} + a_{n-2}x^{n-2} + \ldots + a_2x^2 + a_1x + a_0$,
where a_n is the leading coefficient; a_0 is the constant; and the degree of the polynomial, n, is the exponent of the greatest power of the variable, x.

a) The function $h(x) = 2 - \sqrt{x}$ is a radical function, not a polynomial function.
$\sqrt{x}$ is the same as $x^{\frac{1}{2}}$, which has an exponent that is not a whole number.

b) The function $y = 3x + 1$ is of the form $y = a_1x + a_0$.
It is a polynomial of degree 1. The leading coefficient is 3 and the constant term is 1.

c) The function $f(x) = 3^x$ is not a polynomial function.
The variable x is the exponent.

d) The function $g(x) = 3x^4 - 7$ is of the form $g(x) = a_4x^4 + a_3x^3 + a_2x^2 + a_1x + a_0$.
It is a polynomial of degree 4. The leading coefficient is 3 and the constant term is –7.

e) The function $p(x) = x^{-3} + x^2 + 3x$ is not a polynomial function.
The term x^{-3} has an exponent that is not a whole number.

f) The function $y = -4x^3 + 2x + 5$ is of the form $g(x) = a_3x^3 + a_2x^2 + a_1x + a_0$.
It is a polynomial of degree 3. The leading coefficient is –4 and the constant term is 5.

Section 3.1 Page 114 Question 2

a) The function $f(x) = -x + 3$ has degree 1; it is a linear function with a leading coefficient of –1, and a constant term of 3.

b) The function $y = 9x^2$ has degree 2; it is a quadratic function with a leading coefficient of 9, and a constant term of 0.

c) The function $g(x) = 3x^4 + 3x^3 - 2x + 1$ has degree 4; it is a quartic function with a leading coefficient of 3, and a constant term of 1.

d) First rewrite $k(x) = 4 - 3x^3$ in descending powers of x: $k(x) = -3x^3 + 4$.
The function $k(x) = -3x^3 + 4$ has degree 3; it is a cubic function with a leading coefficient of –3, and a constant term of 4.

e) The function $y = -2x^5 - 2x^3 + 9$ has degree 5; it is a quintic function with a leading coefficient of –2, and a constant term of 9.

f) The function $h(x) = –6$ has degree 0; it is a constant function with a leading coefficient of 0, and a constant term of –6.

Section 3.1 Page 114 Question 3

a) Since the graph of the function extends down into quadrant III and up into quadrant I, it is an odd-degree polynomial function with a positive leading coefficient.
The graph has three x-intercepts. Its domain is $\{x \mid x \in R\}$ and its range is $\{y \mid y \in R\}$.

b) Since the graph of the function extends down into quadrant III and up into quadrant I, it is an odd-degree polynomial function with a positive leading coefficient.
The graph has five x-intercepts. Its domain is $\{x \mid x \in R\}$ and its range is $\{y \mid y \in R\}$.

c) Since the graph of the function opens downward, extending down into quadrant III and down into quadrant IV, it is an even-degree polynomial function with a negative leading coefficient. The graph has three x-intercepts. Its domain is $\{x \mid x \in R\}$ and its range is $\{y \mid y \leq 16.9, y \in R\}$.

d) Since the graph of the function opens downward, extending down into quadrant III and down into quadrant IV, it is an even-degree polynomial function with a negative leading coefficient. The graph has no x-intercepts. Its domain is $\{x \mid x \in R\}$ and its range is $\{y \mid y \leq -3, y \in R\}$.

Section 3.1 Page 114 Question 4

a) The function $f(x) = x^2 + 3x - 1$ is a quadratic (degree 2), which is an even-degree polynomial function. Its graph has a maximum of two x-intercepts. Since the leading coefficient is positive, the graph of the function opens upward, extending up into quadrant II and up into quadrant I, and has a minimum value. The graph has a y-intercept of –1.

b) The function $g(x) = -4x^3 + 2x^2 - x + 5$ is a cubic (degree 3), which is an odd-degree polynomial function. Its graph has at least one x-intercept and at most three x-intercepts. Since the leading coefficient is negative, the graph of the function extends up into quadrant II and down into quadrant IV. The graph has no maximum or minimum values. The graph has a y-intercept of 5.

c) The function $h(x) = -7x^4 + 2x^3 - 3x^2 + 6x + 4$ is a quartic (degree 4), which is an even-degree polynomial function. Its graph has a maximum of four x-intercepts. Since the leading coefficient is negative, the graph of the function opens downward, extending down into quadrant III and down into quadrant IV, and has a maximum value. The graph has a y-intercept of 4.

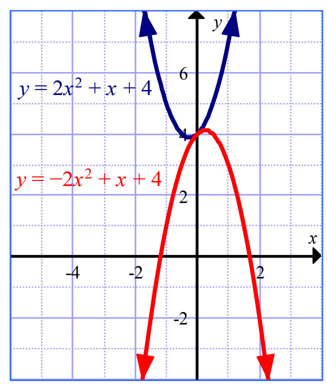
d) The function $q(x) = x^5 - 3x^2 + 9x$ is a quintic (degree 5), which is an odd-degree polynomial function. Its graph has at least one x-intercept and at most five x-intercepts.

Since the leading coefficient is positive, the graph of the function extends down into quadrant III and up into quadrant I. The graph has no maximum or minimum values. The graph has a y-intercept of 0.

e) First rewrite $p(x) = 4 - 2x$ in descending powers of x: $p(x) = -2x + 4$.
The function $p(x) = -2x + 4$ is linear (degree 1), which is an odd-degree polynomial function. Its graph has one x-intercept. Since the leading coefficient is negative, the graph of the function extends up into quadrant II and down into quadrant IV. The graph has no maximum or minimum values. The graph has a y-intercept of 4.

e) First rewrite $v(x) = -x^3 + 2x^4 - 4x^2$ in descending powers of x: $v(x) = 2x^4 - x^3 - 4x^2$.
The function $v(x) = 2x^4 - x^3 - 4x^2$ is a quartic (degree 4), which is an even-degree polynomial function. Its graph has a maximum of four x-intercepts. Since the leading coefficient is positive, the graph of the function opens upward, extending up into quadrant II and up into quadrant I, and has a minimum value. The graph has a y-intercept of 0.

Section 3.1 Page 115 Question 5



Example: I disagree with Jake. Graphs of polynomial functions of the form $y = ax^n + x + b$, where a, b, and n are even integers, will extend from quadrant II to quadrant I only if a and n are positive even integers.
For example, the graph of $y = 2x^2 + x + 4$ has this shape, while the graph of $y = -2x^2 + x + 4$ does not. Also, if n is negative, then the function is no longer a polynomial.

Section 3.1 Page 115 Question 6

Rewrite $P(x) = 1000x + x^4 - 3000$ in descending powers of x: $P(x) = x^4 + 1000x - 3000$.
a) The function $P(x) = x^4 + 1000x - 3000$ has degree 4.

b) The leading coefficient is 1 and the constant is –3000. The constant represents a loss of \$3000 if no snowboards are sold.

c) The function is an even-degree polynomial function. Since the leading coefficient is positive, the graph of the function opens upward, extending up into quadrant II and up into quadrant I.

d) The domain is $\{x \mid x \geq 0, x \in \mathrm{R}\}$. Since x represents the number of snowboards sold, it must be non-negative.

e) The x-intercepts of the graph represent when profit is zero.

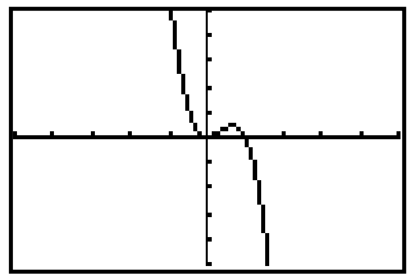
f) For 1500 snowboards, substitute $x = 15$.
$P(x) = 1000x + x^4 - 3000$
$P(15) = 1000(15) + (15)^4 - 3000$
$P(15) = 62\ 625$
The profit from the sale of 1500 snowboards is \$62 625.

Section 3.1 Page 115 Question 7

a) The function $r(d) = -3d^3 + 3d^2$ is a cubic (degree 3).

b) The leading coefficient is –3 and the constant is 0.

c) Its graph has at least one x-intercept and at most three x-intercepts. Since the leading coefficient is negative, the graph of the function extends up into quadrant II and down into quadrant IV (similar to the line $y = -x$).

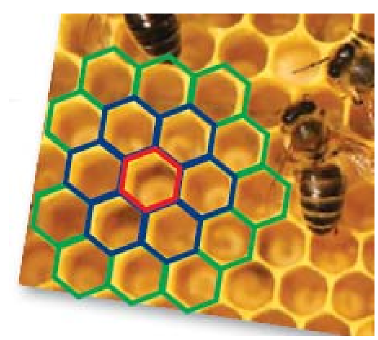


d) The domain is $\{d \mid 0 \le d \le 1, d \in R\}$. Since d represents the amount of drug absorbed into the patient's bloodstream, it must be non-negative and the reaction time must be greater than or equal to zero.

Section 3.1 Page 115 Question 8

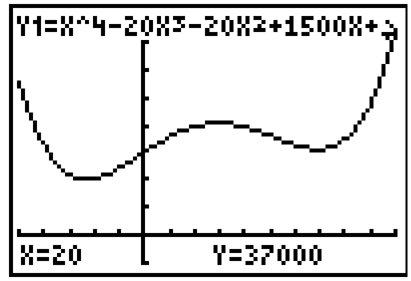
a)

Number of Rings r	**Total Number of Hexagons** $f(r) = 3r^2 - 3r + 1$
1	1
2	7
3	19



b) Substitute $r = 12$ into $f(r) = 3r^2 - 3r + 1$.
$f(12) = 3(12)^2 - 3(12) + 1$
$f(12) = 397$
The total number of hexagons in a honeycomb with 12 rings is 397.

Section 3.1 Page 116 Question 9

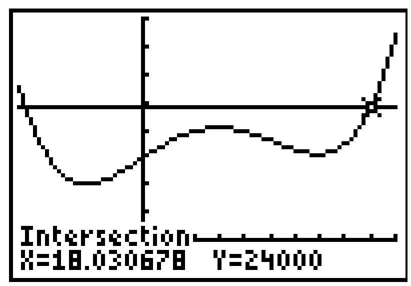


a) The function $P(t) = t^4 - 20t^3 - 20t^2 + 1500t + 15\ 000$ is a quartic (degree 4), which is an even-degree polynomial function. This graph has no x-intercepts. Since the leading coefficient is positive, the graph of the function opens upward, extending up into quadrant II and up into quadrant I, and has a minimum value. The graph has a y-intercept of 15 000. For this situation, the domain is $\{t \mid 0 \le t \le 20, t \in \mathrm{R}\}$ and its range is $\{P \mid 15\ 000 \le P \le 37\ 000, P \in \mathrm{R}\}$.

b) Currently, at $t = 0$, the population is 15 000 people.

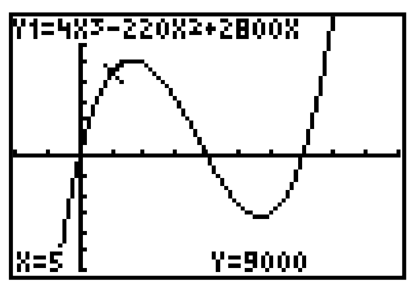
c) Substitute $t = 10$.
$P(t) = t^4 - 20t^3 - 20t^2 + 1500t + 15\ 000$
$P(10) = (10)^4 - 20(10)^3 - 20(10)^2 + 1500(10) + 15\ 000$
$P(10) = 18\ 000$
The population of the town 10 years from now will be 18 000 people.

d) The population of the town be approximately 24 000 in 18 years from now.



Section 3.1 Page 116 Question 10

a) For this situation, the domain is $\{x \mid 0 \le x \le 20 \text{ and } x \ge 35, x \in \mathrm{R}\}$ since x, the height of each box, cannot be negative.

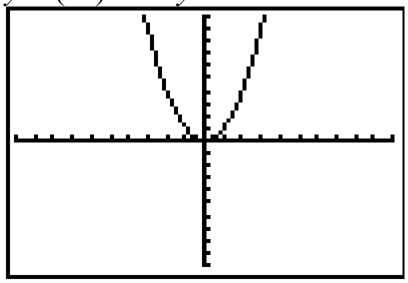


b) $V(x) = 4x^3 - 220x^2 + 2800x$
$V(x) = 4x(x^2 - 55x + 700)$
$V(x) = 4x(x - 20)(x - 35)$
The factored form of the function shows that the zeros of the function are the x-intercepts of its graph.

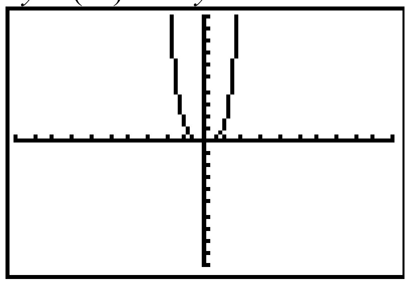
Section 3.1 Page 116 Question 11

a) In each case, the pair of graphs appears to be the same.

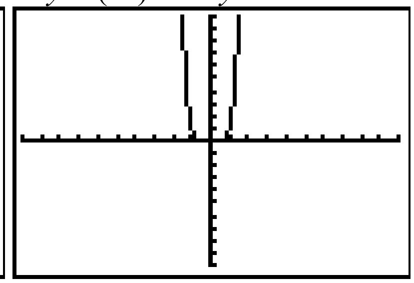
$y = (-x)^2$ and $y = x^2$



$y = (-x)^4$ and $y = x^4$



$y = (-x)^6$ and $y = x^6$

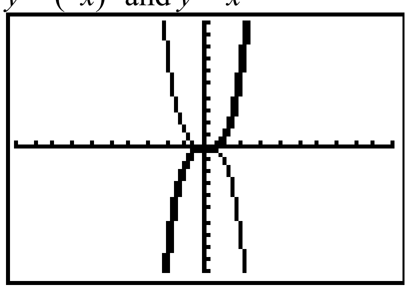


Let n represent a whole number, then $2n$ represents an even whole number.

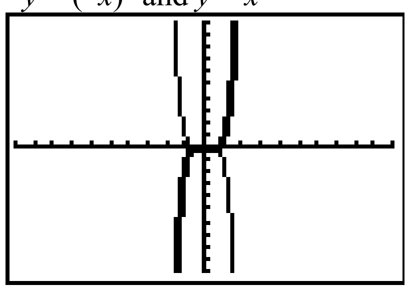
$y = (-x)^{2n}$
$y = (-1)^{2n}x^{2n}$
$y = 1^n x^{2n}$
$y = x^{2n}$

b) In each case, the graphs appear to be reflections of each other in the y-axis (or x-axis).

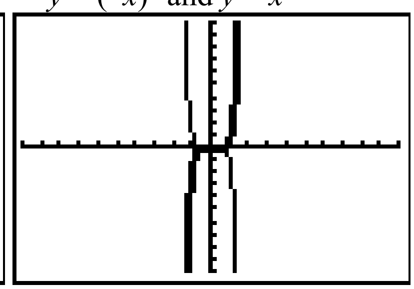
$y = (-x)^3$ and $y = x^3$



$y = (-x)^5$ and $y = x^5$



$y = (-x)^7$ and $y = x^7$



Let n represent a whole number, then $2n + 1$ represents an odd whole number.

$y = (-x)^{2n+1}$
$y = (-1)^{2n+1}x^{2n+1}$
$y = (-1)^{2n}(-1)^1x^{2n+1}$
$y = -(1)^n x^{2n+1}$
$y = -x^{2n+1}$

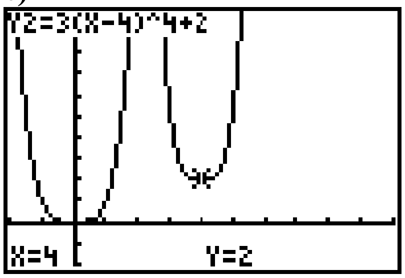
c) For even whole numbers, the graph of the functions are unchanged. For odd whole numbers, the graph of the functions are reflected in the y-axis (or x-axis).

Section 3.1 Page 116 Question 12

a) To obtain the graph of $y = 3(x - 4)^2 + 2$, the graph of $y = x^2$ is stretched vertically by a factor of 3 and translated 4 units to the right and 2 units up.

b) I predict that to obtain the graph of $y = 3(x - 4)^4 + 2$, the graph of $y = x^4$ is stretched vertically by a factor of 3 and translated 4 units to the right and 2 units up.

c)



Section 3.1 Page 116 Question 13

Let a represent the root. If a polynomial equation of degree n has exactly one real root, the corresponding polynomial function is of the form $y = (x - a)^n$. If this is an even-degree function, then its graph will only touch the x-axis. If this is an odd-degree function, then its graph will cross the x-axis once.

Section 3.1 Page 116 Question C1

Let n represent the degree of the polynomial function. Let a represent the maximum or minimum value.

Characteristic	Odd Degree	Even Degree
Number of x-intercepts	minimum of 1 and maximum of n	maximum of n
Number of maximum and minimum points	none	minimum point when $a_n > 0$, maximum point when $a_n < 0$
Domain	$\{x \mid x \in R\}$	$\{x \mid x \in R\}$
Range	$\{y \mid y \in R\}$	$\{y \mid y \geq a, y \in R\}$, $\{y \mid y \leq a, y \in R\}$

Section 3.1 Page 116 Question C2

a) i) A graph that extends from quadrant III to I is an odd-degree function with a positive leading coefficient. For example, $y = x^3$ or $y = 3x^3 + 5$.

ii) A graph that extends from quadrant II to I is an even-degree function with a positive leading coefficient. For example, $y = x^4$ or $y = 2x^4 + 1$.

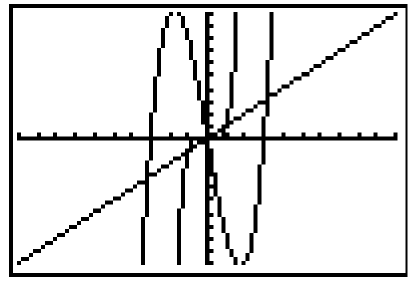
iii) A graph that extends from quadrant II to IV is an odd-degree function with a negative leading coefficient. For example, $y = -x^3$ or $y = -3x^3 + 5$.

iv) A graph that extends from quadrant III to IV is an even-degree function with a negative leading coefficient. For example, $y = -x^4$ or $y = -2x^4 + 1$.

b) Answers should agree on even or odd degree and on the sign of the leading coefficient.

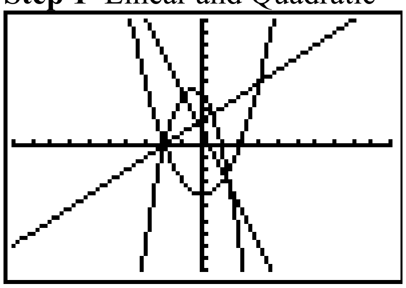
Section 3.1 Page 116 Question C3

Example: The line $y = x$ and polynomial functions with odd degree greater than one and positive leading coefficient extend from quadrant III to quadrant I. Both have no maximum or minimum value. Both have the same domain and range. While the graph of $y = x$ has only one x-intercept, the graph of an odd-degree polynomial function can have one, two, or three x-intercepts.

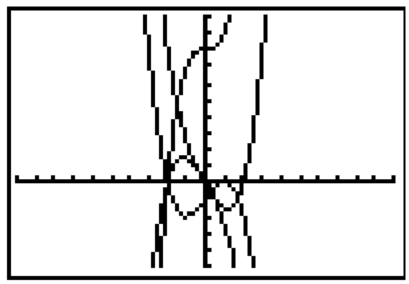


Section 3.1 Page 117 Question C4

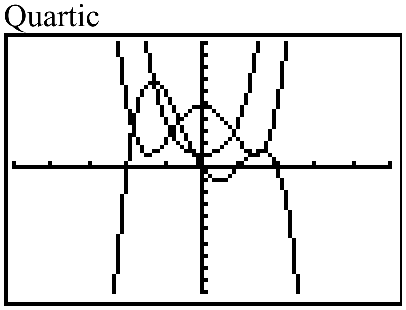
Step 1 Linear and Quadratic



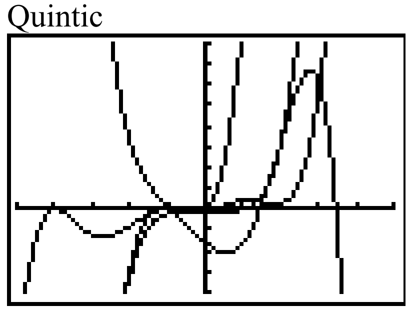
Cubic



Quartic



Quintic



Function	Degree	End Behaviour
$y = x + 2$	1	extends from quadrant III to I
$y = -3x + 1$	1	extends from quadrant II to IV
$y = x^2 - 4$	2	opens upward
$y = -2x^2 - 2x + 4$	2	opens downward
$y = x^3 - 4x$	3	extends from quadrant III to I
$y = -x^3 + 3x - 2$	3	extends from quadrant II to IV
$y = 2x^3 + 16$	3	extends from quadrant III to I
$y = -x^3 - 4x$	3	extends from quadrant II to IV
$y = x^4 - 4x^2 + 5$	4	opens upward
$y = -x^4 + x^3 + 4x^2 - 4x$	4	opens downward
$y = x^4 + 2x^2 + 1$	4	opens upward
$y = x^5 - 2x^4 - 3x^3 + 5x^2 + 4x - 1$	5	extends from quadrant III to I
$y = x^5 - 1$	5	extends from quadrant III to I

$y = -x^5 + x^4 + 8x^3 + 8x^2 - 16x - 16$	5	extends from quadrant II to IV
$y = x(x+1)^2(x+4)^2$	5	extends from quadrant III to I

Step 2 Even-degree functions: a positive leading coefficient means the graph opens upward and a negative leading coefficient means the graph opens downward. Odd-degree functions: a positive leading coefficient means the graph extends from quadrant III to I and a negative leading coefficient means the graph extends from quadrant II to IV.

Step 3 The end behaviours of even-degree functions are always headed in the same direction, either opening upward or downward.

Step 4. The end behaviours of odd-degree functions are always headed away from each other, either extending from quadrant III to I or from quadrant II to IV.

Section 3.2 The Remainder Theorem

Section 3.2 Page 124 Question 1

a)

$$\begin{array}{r}
x+12 \\
x-2\overline{)x^2+10x-24} \\
\underline{x^2-2x} \\
12x-24 \\
\underline{12x-24} \\
0
\end{array}$$

$$\frac{x^2+10x-24}{x-2} = x+12+\frac{0}{x-2}$$

b) The restriction is $x \neq 2$.

c) The corresponding statement that can be used to check the division is $(x-2)(x+12) + 0 = x^2 + 10x - 24$.

d) $(x-2)(x+12) + 0$
$= x^2 + 10x - 24 + 0$
$= x^2 + 10x - 24$

Section 3.2 Page 124 Question 2

a)

$$\begin{array}{l}
\qquad\qquad 3x^3 - 7x^2 + x + 16 \\
x+1\overline{)3x^4 - 4x^3 - 6x^2 + 17x - 8} \\
\quad\ \underline{3x^4 + 3x^3} \\
\qquad\quad -7x^3 - 6x^2 \\
\qquad\quad \underline{-7x^3 - 7x^2} \\
\qquad\qquad\qquad x^2 + 17x \\
\qquad\qquad\qquad \underline{x^2 + x} \\
\qquad\qquad\qquad\quad 16x - 8 \\
\qquad\qquad\qquad\quad \underline{16x + 16} \\
\qquad\qquad\qquad\qquad -24
\end{array}$$

$$\frac{3x^4 - 4x^3 - 6x^2 + 17x - 8}{x+1} = 3x^3 - 7x^2 + x + 16 + \left(\frac{-24}{x+1}\right)$$

b) The restriction is $x \neq -1$.

c) The corresponding statement that can be used to check the division is $(x + 1)(3x^3 - 7x^2 + x + 16) - 24 = 3x^4 - 4x^3 - 6x^2 + 17x - 8$.

d) $(x + 1)(3x^3 - 7x^2 + x + 16) - 24$
$= 3x^4 - 7x^3 + x^2 + 16x + 3x^3 - 7x^2 + x + 16 - 24$
$= 3x^4 - 4x^3 - 6x^2 + 17x - 8$

Section 3.2 Page 124 Question 3

a) For $(x^3 + 3x^2 - 3x - 2) \div (x - 1)$, the quotient, Q, is $x^2 + 4x + 1$.

$$\begin{array}{l}
\qquad\qquad x^2 + 4x + 1 \\
x-1\overline{)x^3 + 3x^2 - 3x - 2} \\
\quad\ \underline{x^3 - x^2} \\
\qquad\ 4x^2 - 3x \\
\qquad\ \underline{4x^2 - 4x} \\
\qquad\qquad\quad x - 2 \\
\qquad\qquad\quad \underline{x - 1} \\
\qquad\qquad\qquad -1
\end{array}$$

b) For $\dfrac{x^3 + 2x^2 - 7x - 2}{x - 2}$, the quotient, Q, is $x^2 + 4x + 1$.

$$\begin{array}{l}
\qquad\qquad x^2 + 4x + 1 \\
x-2\overline{)x^3 + 2x^2 - 7x - 2} \\
\quad\ \underline{x^3 - 2x^2} \\
\qquad\ 4x^2 - 7x \\
\qquad\ \underline{4x^2 - 8x} \\
\qquad\qquad\quad x - 2 \\
\qquad\qquad\quad \underline{x - 2} \\
\qquad\qquad\qquad 0
\end{array}$$

c) For $(2w^3 + 3w^2 - 5w + 2) \div (w + 3)$, the quotient, Q, is $2w^2 - 3w - 4$.

$$\begin{array}{r@{}l}
 & 2w^2 - 3w - 4 \\
w+3 & \overline{)2w^3 + 3w^2 - 5w + 2} \\
 & \underline{2w^3 + 6w^2} \\
 & \qquad -3w^2 - 5w \\
 & \qquad \underline{-3w^2 - 9w} \\
 & \qquad\qquad -4w + 2 \\
 & \qquad\qquad \underline{-4w - 12} \\
 & \qquad\qquad\qquad 14
\end{array}$$

d) For $(9m^3 - 6m^2 + 3m + 2) \div (m - 1)$, the quotient, Q, is $9m^2 + 3m + 6$.

$$\begin{array}{r@{}l}
 & 9m^2 + 3m + 6 \\
m-1 & \overline{)9m^3 - 6m^2 + 3m + 2} \\
 & \underline{9m^3 - 9m^2} \\
 & \qquad 3m^2 + 3m \\
 & \qquad \underline{3m^2 - 3m} \\
 & \qquad\qquad 6m + 2 \\
 & \qquad\qquad \underline{6m - 6} \\
 & \qquad\qquad\qquad 8
\end{array}$$

e) For $\dfrac{t^4 + 6t^3 - 3t^2 - t + 8}{t + 1}$, the quotient, Q, is $t^3 + 5t^2 - 8t + 7$.

$$\begin{array}{r@{}l}
 & t^3 + 5t^2 - 8t + 7 \\
t+1 & \overline{)t^4 + 6t^3 - 3t^2 - t + 8} \\
 & \underline{t^4 + t^3} \\
 & \qquad 5t^3 - 3t^2 \\
 & \qquad \underline{5t^3 + 5t^2} \\
 & \qquad\qquad -8t^2 - t \\
 & \qquad\qquad \underline{-8t^2 - 8t} \\
 & \qquad\qquad\qquad 7t + 8 \\
 & \qquad\qquad\qquad \underline{7t + 7} \\
 & \qquad\qquad\qquad\qquad 1
\end{array}$$

f) For $(2y^4 - 3y^2 + 1) \div (y - 3)$, the quotient, Q, is $2y^3 + 6y^2 + 15y + 45$.

$$\begin{array}{r@{}l}
 & 2y^3 + 6y^2 + 15y + 45 \\
y-3 & \overline{)2y^4 + 0y^3 - 3y^2 + 0y + 1} \\
 & \underline{2y^4 - 6y^3} \\
 & \qquad 6y^3 - 3y^2 \\
 & \qquad \underline{6y^3 - 18y^2} \\
 & \qquad\qquad 15y^2 + 0y \\
 & \qquad\qquad \underline{15y^2 - 45y} \\
 & \qquad\qquad\qquad 45y + 1 \\
 & \qquad\qquad\qquad \underline{45y - 135} \\
 & \qquad\qquad\qquad\qquad 136
\end{array}$$

Section 3.2 Page 124 Question 4

a) For $(x^3 + x^2 + 3) \div (x + 4)$, the quotient, Q, is $x^2 - 3x + 12$.

+4	1	1	0	1
–		4	–12	48
×	1	–3	12	–47

b) For $\dfrac{m^4 - 2m^3 + m^2 + 12m - 6}{m - 2}$, the quotient, Q, is $m^3 + m + 14$.

–2	1	–2	1	12	–6
–		–2	0	–2	–28
×	1	0	1	14	22

c) For $(2 - x + x^2 - x^3 - x^4) \div (x + 2)$, the quotient, Q, is $-x^3 + x^2 - x + 1$.

+2	–1	–1	1	–1	2
–		–2	2	–2	2
×	–1	1	–1	1	0

d) For $(2s^3 + 3s^2 - 9s - 10) \div (s - 2)$, the quotient, Q, is $2s^2 + 7s + 5$.

–2	2	3	–9	–10
–		–4	–14	–10
×	2	7	5	0

e) For $\dfrac{h^3 + 2h^2 - 3h + 9}{h + 3}$, the quotient, Q, is $h^2 - h$.

+3	1	2	–3	9
–		3	–3	0
×	1	–1	0	9

f) For $(2x^3 + 7x^2 - x + 1) \div (x + 2)$, the quotient, Q, is $2x^2 + 3x - 7$.

+2	2	7	–1	1
–		4	6	–14
×	2	3	–7	15

Section 3.2 Page 124 Question 5

a) For $(x^3 + 7x^2 - 3x + 4) \div (x + 2)$, $x \neq -2$.

+2	1	7	–3	4
–		2	10	–26
×	1	5	–13	30

$$\frac{x^3 + 7x^2 - 3x + 4}{x + 2} = x^2 + 5x - 13 + \frac{30}{x + 2}$$

b) For $\dfrac{11t - 4t^4 - 7}{t - 3}$, $t \neq 3$.

–3	–4	0	0	11	–7
–		12	36	108	291
×	–4	–12	–36	–97	–298

$$\frac{11t - 4t^4 - 7}{t - 3} = -4t^3 - 12t^2 - 36t - 97 - \frac{298}{t - 3}$$

c) For $(x^3 + 3x^2 - 2x + 5) \div (x + 1)$, $x \neq -1$.

+1	1	3	–2	5
–		1	2	–4
×	1	2	–4	9

$$\frac{x^3 + 3x^2 - 2x + 5}{x + 1} = x^2 + 2x - 4 + \frac{9}{x + 1}$$

d) For $(4n^2 + 7n - 5) \div (n + 3)$, $n \neq -3$.

+3	4	7	–5
–		12	–15
×	4	–5	10

$$\frac{4n^2 - 7n - 5}{n + 3} = 4n - 5 + \frac{10}{n + 3}$$

e) For $\dfrac{4n^3 - 15n + 2}{n - 3}$, $n \neq 3$.

–3	4	0	–15	2
–		–12	–36	–63
×	4	12	21	65

$$\frac{4n^3 - 15n + 2}{n - 3} = 4n^2 + 12n + 21 + \frac{65}{n - 3}$$

f) For $(x^3 + 6x^2 - 4x + 1) \div (x + 2)$, $x \neq -2$.

+2	1	6	–4	1
–		2	8	–24
×	1	4	–12	25

$$\frac{x^3 + 6x^2 - 4x + 1}{x + 2} = x^2 + 4x - 12 + \frac{25}{x + 2}$$

Section 3.2 Page 124 Question 6

Evaluate $P(–2)$.

a) $P(x) = x^3 + 3x^2 - 5x + 2$
$P(-2) = (-2)^3 + 3(-2)^2 - 5(-2) + 2$
$P(-2) = -8 + 12 + 10 + 2$
$P(-2) = 16$
The remainder when $x^3 + 3x^2 - 5x + 2$ is divided by $x + 2$ is 16.

b) $P(x) = 2x^4 - 2x^3 + 5x$
$P(-2) = 2(-2)^4 - 2(-2)^3 + 5(-2)$
$P(-2) = 32 + 16 - 10$
$P(-2) = 38$
The remainder when $2x^4 - 2x^3 + 5x$ is divided by $x + 2$ is 38.

c) $P(x) = x^4 + x^3 - 5x^2 + 2x - 7$
$P(-2) = (-2)^4 + (-2)^3 - 5(-2)^2 + 2(-2) - 7$
$P(-2) = 16 - 8 - 20 - 4 - 7$
$P(-2) = -23$
The remainder when $x^4 + x^3 - 5x^2 + 2x - 7$ is divided by $x + 2$ is –23.

d) $P(x) = 8x^3 + 4x^2 - 19$
$P(-2) = 8(-2)^3 + 4(-2)^2 - 19$
$P(-2) = -64 + 16 - 19$
$P(-2) = -67$
The remainder when $8x^3 + 4x^2 - 19$ is divided by $x + 2$ is –67.

e) $P(x) = 3x^3 - 12x - 2$
$P(-2) = 3(-2)^3 - 12(-2) - 2$
$P(-2) = -24 + 24 - 2$
$P(-2) = -2$
The remainder when $3x^3 - 12x - 2$ is divided by $x + 2$ is –2.

f) $P(x) = 2x^3 + 3x^2 - 5x + 2$
$P(-2) = 2(-2)^3 + 3(-2)^2 - 5(-2) + 2$
$P(-2) = -16 + 12 + 10 + 2$
$P(-2) = 8$
The remainder when $2x^3 + 3x^2 - 5x + 2$ is divided by $x + 2$ is 8.

Section 3.2 Page 124 Question 7

a) Evaluate $P(–3)$.
$P(x) = x^3 + 2x^2 - 3x + 9$
$P(-3) = (-3)^3 + 2(-3)^2 - 3(-3) + 9$
$P(-3) = -27 + 18 + 9 + 9$
$P(-3) = 9$
The remainder when $x^3 + 2x^2 - 3x + 9$ is divided by $x + 3$ is 9.

b) Evaluate $P(2)$.
$P(t) = 2t - 4t^3 - 3t^2$
$P(2) = 2(2) - 4(2)^3 - 3(2)^2$
$P(2) = 4 - 32 - 12$
$P(2) = -40$
The remainder when $2t - 4t^3 - 3t^2$ is divided by $t - 2$ is -40.

c) Evaluate $P(3)$.
$P(x) = x^3 + 2x^2 - 3x + 5$
$P(3) = (3)^3 + 2(3)^2 - 3(3) + 5$
$P(3) = 27 + 18 - 9 + 5$
$P(3) = 41$
The remainder when $x^3 + 2x^2 - 3x + 5$ is divided by $x - 3$ is 41.

d) Evaluate $P(2)$.
$P(n) = n^4 - 3n^2 - 5n + 2$
$P(2) = (2)^4 - 3(2)^2 - 5(2) + 2$
$P(3) = 16 - 12 - 10 + 2$
$P(3) = -4$
The remainder when $n^4 - 3n^2 - 5n + 2$ is divided by $n - 2$ is -4.

Section 3.2 Page 124 Question 8

a) Solve $P(1) = 3$ to determine the value of k.
$P(x) = x^3 + 4x^2 - x + k$
$3 = 1^3 + 4(1)^2 - 1 + k$
$3 = 1 + 4 - 1 + k$
$k = -1$

b) Solve $P(2) = 3$ to determine the value of k.
$P(x) = x^3 + x^2 + kx - 15$
$3 = 2^3 + 2^2 + k(2) - 15$
$3 = 8 + 4 + 2k - 15$
$k = 3$

c) Solve $P(-2) = 3$ to determine the value of k.
$P(x) = x^3 + kx^2 + x + 5$
$3 = (-2)^3 + k(-2)^2 + (-2) + 5$
$3 = -8 + 4k - 2 + 5$
$k = 2$

d) Solve $P(-2) = 3$ to determine the value of k.

$P(x) = kx^3 + 3x + 1$

$3 = k(-2)^3 + 3(-2) + 1$

$3 = -8k - 6 + 1$

$k = -1$

Section 3.2 Page 124 Question 9

First, determine an expression for $P(2)$ and $P(-1)$.

$P(x) = -2x^3 + cx^2 - 5x + 2$

$P(2) = -2(2)^3 + c(2)^2 - 5(2) + 2$

$P(2) = -16 + 4c - 10 + 2$

$P(2) = 4c - 24$

$P(x) = -2x^3 + cx^2 - 5x + 2$

$P(-1) = -2(-1)^3 + c(-1)^2 - 5(-1) + 2$

$P(-1) = 2 + c + 5 + 2$

$P(-1) = c + 9$

Then, solve $P(2) = P(-1)$ to determine the value of c.

$4c - 24 = c + 9$

$3c = 33$

$c = 11$

Section 3.2 Page 125 Question 10

Solve $P(-k) = 14$ to determine the value of k.

$P(x) = 3x^2 + 6x - 10$

$14 = 3(-k)^2 + 6(-k) - 10$

$14 = 3k^2 - 6k - 10$

$0 = 3k^2 - 6k - 24$

$0 = k^2 - 2k - 8$

$0 = (k - 4)(k + 2)$

$k - 4 = 0$ or $k + 2 = 0$

$k = 4$ $\quad$ $k = -2$

Section 3.2 Page 125 Question 11

a) $A(x) = 2x^2 - x - 6$

$A(x) = (2x + 3)(x - 2)$

If the height of the rectangle is $x - 2$, then the width is $2x + 3$.

b) Evaluate $A(3)$.

$A(x) = 2x^2 - x - 6$

$A(3) = 2(3)^2 - (3) - 6$

$A(3) = 18 - 3 - 6$

$A(3) = 9$

The remainder is 9. This represents a portion of the width: $\dfrac{2x^2 - x - 6}{x - 3} = 2x + 5 + \dfrac{9}{x - 3}$.

Section 3.2 Page 125 Question 12

a) Determine $2n^2 - 4n + 3 \div n - 3$.

$$\begin{array}{r|rrr} -3 & 2 & -4 & 3 \\ - & & -6 & -6 \\ \hline \times & 2 & 2 & 9 \end{array}$$

An expression for the other real number is $2n + 2 + \dfrac{9}{n-3}$.

b) For $n = 1$:

$$\begin{aligned} n - 3 &= 1 - 3 \\ &= -2 \end{aligned}$$

$$\begin{aligned} 2n + 2 + \frac{9}{n-3} &= 2(1) + 2 + \frac{9}{1-3} \\ &= 2 + 2 - 4.5 \\ &= -0.5 \end{aligned}$$

For $n = 1$, the two numbers are –2 and –0.5.

Section 3.2 Page 125 Question 13

a) Use synthetic division.

$$\begin{array}{r|rrrr} +3 & 9\pi & 51\pi & 88\pi & 48\pi \\ - & & 27\pi & 72\pi & 48\pi \\ \hline \times & 9\pi & 24\pi & 16\pi & 0 \end{array}$$

The expression $9\pi x^2 + 24\pi x + 16\pi$ represents the area of the base of the cylindrical containers.

b) $V(x) = (9\pi x^2 + 24\pi x + 16\pi)(x + 3)$
$V(x) = \pi(9x^2 + 24x + 16)(x + 3)$
$V(x) = \pi(3x + 4)^2(x + 3)$
$V = \pi r^2 h$

In this factored form, $3x + 4$ represents the radius, r, and $x + 3$ represents the height, h.

c) If $2 \le x \le 8$, then $10 \le r \le 28$ and $5 \le h \le 11$.
For the given values of x, the radius can be from 10 cm to 28 cm and the height can be from 5 cm to 11 cm.

Section 3.2 Page 125 Question 14

Solve a system of equations represented by $P(-3) = -1$ and $P(2) = -4$.

$$\begin{aligned} P(x) &= mx^3 - 3x^2 + nx + 2 \\ -1 &= m(-3)^3 - 3(-3)^2 + n(-3) + 2 \\ -1 &= -27m - 27 - 3n + 2 \\ 24 &= -27m - 3n \\ 8 &= -9m - n \quad ① \end{aligned}$$

$$\begin{aligned} P(x) &= mx^3 - 3x^2 + nx + 2 \\ -4 &= m(2)^3 - 3(2)^2 + n(2) + 2 \\ -4 &= 8m - 12 + 2n + 2 \\ 6 &= 8m + 2n \\ 3 &= 4m + n \quad ② \end{aligned}$$

Add equations ① and ②: $11 = -5m$

$$m = -\frac{11}{5}$$

Substitute $m = -\frac{11}{5}$ into equation ②: $3 = 4\left(-\frac{11}{5}\right) + n$

$$n = \frac{59}{5}$$

Section 3.2 Page 125 Question 15

Solve a system of equations represented by $P(2) = -5$ and $P(-1) = -16$.

$$\begin{aligned} P(x) &= 3x^3 + ax^2 + bx - 9 \\ -5 &= 3(2)^3 + a(2)^2 + b(2) - 9 \\ -5 &= 24 + 4a + 2b - 9 \\ -20 &= 4a + 2b \\ -10 &= 2a + b \quad ① \end{aligned}$$

$$\begin{aligned} P(x) &= 3x^3 + ax^2 + bx - 9 \\ -16 &= 3(-1)^3 + a(-1)^2 + b(-1) - 9 \\ -16 &= -3 + a - b - 9 \\ -4 &= a - b \quad ② \end{aligned}$$

Add equations ① and ②: $-14 = 3a$

$$a = -\frac{14}{3}$$

Substitute $a = -\frac{14}{3}$ into equation ②: $-4 = -\frac{14}{3} - b$

$$b = -\frac{2}{3}$$

Section 3.2 Page 125 Question 16

To determine the remainder when $10x^4 - 11x^3 - 8x^2 + 7x + 9$ is divided by $2x - 3$, using synthetic division, rewrite the binomial as $x - \frac{3}{2}$, or $x - 1.5$.

−1.5	10	−11	−8	7	9
−		−15	−6	3	−6
×	10	4	−2	4	15

The remainder is 15.

Section 3.2 Page 125 Question 17

Examples:

a) A quadratic polynomial that gives $P(3) = -4$ is $P(x) = (x - 1)(x - 3) - 4$, or $P(x) = x^2 - 4x - 1$.

b) A cubic polynomial that gives $P(-2) = 4$ is $P(x) = (x - 1)^2(x + 2) + 4$, or $P(x) = x^3 - 3x + 6$.

c) A quartic polynomial that gives $P(0.5) = 1$ is $P(x) = (x - 1)^3(2x - 1) + 1$, or $P(x) = 2x^4 - 7x^3 + 9x^2 - 5x + 2$.

Section 3.2 Page 125 Question C1

Example: The process is basically the same. However in long division of polynomials, additional terms are brought down and added to the previous results. Polynomial division also results in a restriction.

Section 3.2 Page 125 Question C2

a) If the remainder is 0, then $x - a$ is a factor of $bx^2 + cx + d$.

b) Evaluate $P(a)$.

$P(x) = bx^2 + cx + d$
$P(a) = ba^2 + ca + d$
$P(a) = a^2b + ac + d$
The remainder is $a^2b + ac + d$.

Section 3.2 Page 125 Question C3

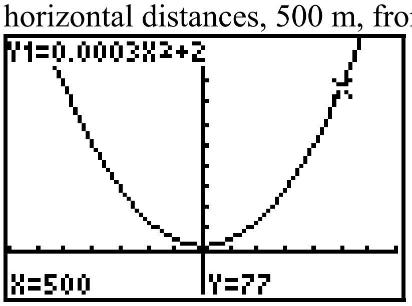
a) Evaluate $h(500)$.

$h(d) = 0.0003d^2 + 2$
$h(500) = 0.0003(500)^2 + 2$
$h(500) = 77$
The remainder is 77.

b) Evaluate $h(–500)$.

$h(d) = 0.0003d^2 + 2$
$h(-500) = 0.0003(-500)^2 + 2$
$h(-500) = 752$
The remainder is 77.

c) The remainders are the same. They represent the height of the cable at the given horizontal distances, 500 m, from the lowest point.



Section 3.3 The Factor Theorem

Section 3.3 Page 133 Question 1

a) If $P(1) = 0$, then a corresponding binomial factor of a polynomial, $P(x)$, is $x – 1$.

b) If $P(–3) = 0$, then a corresponding binomial factor of a polynomial, $P(x)$, is $x + 3$.

c) If $P(4) = 0$, then a corresponding binomial factor of a polynomial, $P(x)$, is $x – 4$.

d) If $P(a) = 0$, then a corresponding binomial factor of a polynomial, $P(x)$, is $x – a$.

Section 3.3 Page 133 Question 2

a) Evaluate $P(1)$.
$P(x) = x^3 - 3x^2 + 4x - 2$
$P(1) = (1)^3 – 3(1)^2 + 4(1) – 2$
$P(1) = 1 – 3 + 4 – 2$
$P(1) = 0$
Since the remainder is zero, $x – 1$ is a factor of $P(x)$.

b) Evaluate $P(1)$.
$P(x) = 2x^3 - x^2 - 3x - 2$
$P(1) = 2(1)^3 – (1)^2 – 3(1) – 2$
$P(1) = 2 – 1 – 3 – 2$
$P(1) = –4$
Since the remainder is not zero, $x – 1$ is not a factor of $P(x)$.

c) Evaluate $P(1)$.
$P(x) = 3x^3 - x - 3$
$P(1) = 3(1)^3 – (1)^2 – 3$
$P(1) = 3 – 1 – 3$
$P(1) = –1$
Since the remainder is not zero, $x – 1$ is not a factor of $P(x)$.

d) Evaluate $P(1)$.
$P(x) = 2x^3 + 4x^2 - 5x - 1$
$P(1) = 2(1)^3 + 4(1)^2 – 5(1) – 1$
$P(1) = 2 + 4 – 5 – 1$
$P(1) = 0$
Since the remainder is zero, $x – 1$ is a factor of $P(x)$.

e) Evaluate $P(1)$.
$P(x) = x^4 - 3x^3 + 2x^2 - x + 1$
$P(1) = (1)^4 – 3(1)^3 + 2(1)^2 – 1 + 1$
$P(1) = 1 – 3 + 2 – 1 + 1$
$P(1) = 0$

Since the remainder is zero, $x - 1$ is a factor of $P(x)$.

f) Evaluate $P(1)$.
$P(x) = 4x^4 - 2x^3 + 3x^2 - 2x + 1$
$P(1) = 4(1)^4 - 2(1)^3 + 3(1)^2 - 2(1) + 1$
$P(1) = 4 - 2 + 3 - 2 + 1$
$P(1) = 4$
Since the remainder is not zero, $x - 1$ is not a factor of $P(x)$.

Section 3.3 Page 133 Question 3

a) Evaluate $P(-2)$.
$P(x) = 5x^2 + 2x + 6$
$P(-2) = 5(-2)^2 + 2(-2) + 6$
$P(-2) = 20 - 4 + 6$
$P(-2) = 22$
Since the remainder is not zero, $x + 2$ is not a factor of $P(x)$.

b) Evaluate $P(-2)$.
$P(x) = 2x^3 - x^2 - 5x - 8$
$P(-2) = 2(-2)^3 - (-2)^2 - 5(-2) - 8$
$P(-2) = -16 - 4 + 10 - 8$
$P(-2) = -18$
Since the remainder is not zero, $x + 2$ is not a factor of $P(x)$.

c) Evaluate $P(-2)$.
$P(x) = 2x^3 + 2x^2 - x - 6$
$P(-2) = 2(-2)^3 + 2(-2)^2 - (-2) - 6$
$P(-2) = -16 + 8 + 2 - 6$
$P(-2) = -12$
Since the remainder is not zero, $x + 2$ is not a factor of $P(x)$.

d) Evaluate $P(-2)$.
$P(x) = x^4 - 2x^2 + 3x - 4$
$P(-2) = (-2)^4 - 2(-2)^2 + 3(-2) - 4$
$P(-2) = 16 - 8 - 6 - 4$
$P(-2) = -2$
Since the remainder is not zero, $x + 2$ is not a factor of $P(x)$.

e) Evaluate $P(-2)$.
$P(x) = x^4 + 3x^3 - x^2 - 3x + 6$
$P(-2) = (-2)^4 + 3(-2)^3 - (-2)^2 - 3(-2) + 6$
$P(-2) = 16 - 24 - 4 + 6 + 6$
$P(-2) = 0$
Since the remainder is zero, $x + 2$ is a factor of $P(x)$.

f) Evaluate $P(–2)$.
$P(x) = 3x^4 + 5x^3 + x - 2$
$P(-2) = 3(-2)^4 + 5(-2)^2 + (-2) - 2$
$P(-2) = 48 + 20 - 2 - 2$
$P(-2) = 64$
Since the remainder is not zero, $x + 2$ is not a factor of $P(x)$.

Section 3.3 Page 133 Question 4

a) Let $P(x) = x^3 + 3x^2 - 6x - 8$. The possible integral zeros of the polynomial are the factors of the constant term, –8: ±1, ±2, ±4, and ±8.

b) Let $P(s) = s^3 + 4s^2 - 15s - 18$. The possible integral zeros of the polynomial are the factors of the constant term, –18: ±1, ±2, ±3, ±6, ±9, and ±18.

c) Let $P(n) = n^3 - 3n^2 - 10n + 24$. The possible integral zeros of the polynomial are the factors of the constant term, 24: ±1, ±2, ±3, ±4, ±6, ±8, ±12, and ±24.

d) Let $P(p) = p^4 - 2p^3 - 8p^2 + 3p - 4$. The possible integral zeros of the polynomial are the factors of the constant term, –4: ±1, ±2, and ±4.

e) Let $P(z) = z^4 + 5z^3 + 2z^2 + 7z - 15$. The possible integral zeros of the polynomial are the factors of the constant term, –15: ±1, ±3, ±5, and ±15.

f) Let $P(y) = y^4 - 5y^3 - 7y^2 + 21y + 4$. The possible integral zeros of the polynomial are the factors of the constant term, 4: ±1, ±2, and ±4.

Section 3.3 Page 134 Question 5

a) For $P(x) = x^3 - 6x^2 + 11x - 6$, the possible integral zeros of the polynomial are the factors of the constant term, –6: ±1, ±2, ±3, and ±6. Test these values to find a first factor: $P(1) = 0$. Use synthetic division to find the other factors.

–1	1	–6	11	–6
–		–1	5	–6
×	1	–5	6	0

Then, the remaining factor $x^2 - 5x + 6$ can be factored as $(x - 2)(x - 3)$.
So, $P(x) = (x - 1)(x - 2)(x - 3)$.

b) For $P(x) = x^3 + 2x^2 - x - 2$, the possible integral zeros of the polynomial are the factors of the constant term, –2: ±1 and ±2. Test these values to find a first factor: $P(1) = 0$. Use synthetic division to find the other factors.

–1	1	2	–1	–2
–		–1	–3	–2
×	1	3	2	0

Then, the remaining factor $x^2 + 3x + 2$ can be factored as $(x + 1)(x + 2)$.
So, $P(x) = (x – 1)(x + 1)(x + 2)$.

c) For $P(v) = v^3 + v^2 - 16v - 16$, the possible integral zeros of the polynomial are the factors of the constant term, –16: ±1, ±2, ±4, ±8, and ±16. Test these values to find a first factor: $P(–1) = 0$. Use synthetic division to find the other factors.

+1	1	1	–16	–16
–		1	0	–16
×	1	0	–16	0

Then, the remaining factor $v^2 – 16$ can be factored as $(v + 4)(v – 4)$.
So, $P(v) = (v + 1)(v + 4)(v – 4)$.

d) For $P(x) = x^4 + 4x^3 - 7x^2 - 34x - 24$, the possible integral zeros of the polynomial are the factors of the constant term, –24: ±1, ±2, ±3, ±4, ±6, ±8, ±12, and ±24. Test these values to find a first factor: $P(–1) = 0$. Use synthetic division to find the other factors.

+1	1	4	–7	–34	–24
–		1	3	–10	–24
×	1	3	–10	–24	0

Repeat the process with the remaining factor $x^3 + 3x^2 – 10x – 24$. Test factors of the constant term, –24: $P(–2) = 0$.

+2	1	3	–10	–24
–		2	2	–24
×	1	1	–12	0

Then, the remaining factor $x^2 + x – 12$ can be factored as $(x + 4)(x – 3)$.
So, $P(x) = (x + 1)(x + 2)(x + 4)(x – 3)$.

e) For $P(k) = k^5 + 3k^4 - 5k^3 - 15k^2 + 4k + 12$, the possible integral zeros of the polynomial are the factors of the constant term, 12: ±1, ±2, ±3, ±4, ±6, and ±12. Test these values to find a first factor: $P(1) = 0$. Use synthetic division to find the other factors.

–1	1	3	–5	–15	4	12
–		–1	–4	1	16	12
×	1	4	–1	–16	–12	0

Repeat the process with the remaining factor $k^4 + 4k^3 – k^2 – 16k – 12$. Test factors of the constant term, –12: $P(–1) = 0$.

+1	1	4	–1	–16	–12
–		1	3	–4	–12
×	1	3	–4	–12	0

Then, the remaining factor $k^3 + 3k^2 – 4k – 12$ can be factored by grouping as $(k + 3)(k^2 – 4)$.
So, $P(k) = (k – 1)(k + 1)(k + 3)(k + 2)(k – 2)$.

Section 3.3 Page 134 Question 6

a) For $P(x) = x^3 - 2x^2 - 9x + 18$, the possible integral zeros of the polynomial are the factors of the constant term, 18: ±1, ±2, ±3, ±6, ±9, and ±18. Test these values to find a first factor: $P(2) = 0$. Use synthetic division to find the other factors.

–2	1	–2	–9	18
–		–2	0	18
×	1	0	–9	0

Then, the remaining factor $x^2 – 9$ can be factored as $(x + 3)(x – 3)$.
So, $P(x) = (x – 2)(x + 3)(x – 3)$.

b) For $P(t) = t^3 + t^2 - 22t - 40$, the possible integral zeros of the polynomial are the factors of the constant term, –40: ±1, ±2, ±4, ±5, ±8, ±10, ±20, and ±40. Test these values to find a first factor: $P(–2) = 0$. Use synthetic division to find the other factors.

+2	1	1	–22	–40
–		2	–2	–40
×	1	–1	–20	0

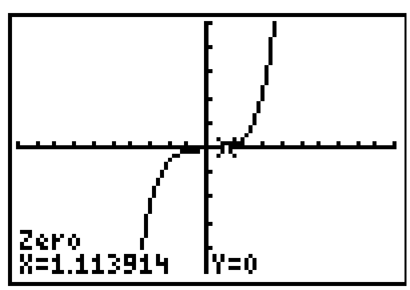
Then, the remaining factor $t^2 – t – 20$ can be factored as $(t + 4)(t – 5)$.
So, $P(t) = (t + 2)(t + 4)(t – 5)$.

c) For $P(h) = h^3 - 27h + 10$, the possible integral zeros of the polynomial are the factors of the constant term, 10: ±1, ±2, ±5, and ±10. Test these values to find a first factor: $P(5) = 0$. Use synthetic division to find the other factors.

–5	1	0	–27	10
–		–5	–25	10
×	1	5	–2	0

Then, the remaining factor $h^2 + 5h – 2$ cannot be factored.
So, $P(h) = (h – 5)(h^2 + 5h – 2)$.

d) For $P(x) = x^5 + 8x^3 + 2x - 15$, the possible integral zeros of the polynomial are the factors of the constant term, –15: ±1, ±3, ±5, and ±15. Test these values to find that none of them is a factor.
Graphing the corresponding function shows one non-integral zero at $x \approx 1.1$.



e) For $P(q) = q^4 + 2q^3 + 2q^2 - 2q - 3$, the possible integral zeros of the polynomial are the factors of the constant term, –3: ±1 and ±3. Test these values to find a first factor: $P(1) = 0$. Use synthetic division to find the other factors.

–1	1	2	2	–2	–3
–		–1	–3	–5	–3
×	1	3	5	3	0

Repeat the process with the remaining factor $q^3 + 3q^2 + 5q + 3$. Test factors of the constant term, 3: $P(–1) = 0$.

+1	1	3	5	3
–		1	2	3
×	1	2	3	0

Then, the remaining factor $q^2 + 2q + 3$ cannot be factored.
So, $P(q) = (q – 1)(q + 1)(q^2 + 2q + 3)$.

Section 3.3 Page 134 Question 7

a) Solve $P(2) = 0$ to determine the value(s) of k.

$P(x) = x^2 – x + k$
$0 = 2^2 – 2 + k$
$0 = 2 + k$
$k = –2$

b) Solve $P(–k) = 0$ to determine the value(s) of k.

$P(x) = x^2 − 6x − 7$
$0 = (–k)^2 – 6(–k) – 7$
$0 = k^2 + 6k – 7$
$0 = (k + 7)(k – 1)$
$k + 7 = 0$ or $k – 1 = 0$
$k = –7$ $\quad$ $k = 1$

c) Solve $P(–2) = 0$ to determine the value(s) of k.

$P(x) = x^3 + 4x^2 + x + k$
$0 = (–2)^3 + 4(–2)^2 + (–2) + k$
$0 = 6 + k$
$k = –6$

d) Solve $P(2) = 0$ to determine the value(s) of k.

$P(x) = x^2 + kx − 16$
$0 = 2^2 + k(2) – 16$
$0 = 2k – 12$
$k = 6$

Section 3.3 Page 134 Question 8

Factor $V(h) = h^3 - 2h^2 + h$.
$V(h) = h^3 - 2h^2 + h$
$V(h) = h(h^2 - 2h + 1)$
$V(h) = h(h - 1)(h - 1)$
The possible dimensions of the bookcase are h, $h - 1$, and $h - 1$.

Section 3.3 Page 134 Question 9

Factor $V(\ell) = \ell^3 - 2\ell^2 - 15\ell$.
$V(\ell) = \ell^3 - 2\ell^2 - 15\ell$
$V(\ell) = \ell(\ell^2 - 2\ell - 15)$
$V(\ell) = \ell(\ell - 5)(\ell + 3)$
The possible width and height of the racquetball court are $\ell - 5$ and $\ell + 3$.

Section 3.3 Page 134 Question 10

For $V(x) = x^3 + 5x^2 - 2x - 24$, the possible integral zeros of the polynomial are the factors of the constant term, –24: ±1, ±2, ±3, ±4, ±6, ±8, ±12, and ±24. Test these values to find a first factor: $P(2) = 0$. Use synthetic division to find the other factors.

$$\begin{array}{r|rrrr} -2 & 1 & 5 & -2 & -24 \\ - & & -2 & -14 & -24 \\ \hline \times & 1 & 7 & 12 & 0 \end{array}$$

Then, the remaining factor $x^2 + 7x + 12$ can be factored as $(x + 3)(x + 4)$.
So, $V(x) = (x - 2)(x + 3)(x + 4)$.
The possible dimensions of the block, in centimetres, are $(x - 2)$ cm by $(x + 3)$ cm by $(x + 4)$ cm.

Section 3.3 Page 134 Question 11

For $V(x) = x^3 + 14x^2 + 63x + 90$, the given factor is $x + 6$. Use synthetic division to find the other factors.

$$\begin{array}{r|rrrr} +6 & 1 & 14 & 63 & 90 \\ - & & 6 & 48 & 90 \\ \hline \times & 1 & 8 & 15 & 0 \end{array}$$

Then, the remaining factor $x^2 + 8x + 15$ can be factored as $(x + 3)(x + 5)$.
So, $V(x) = (x + 6)(x + 3)(x + 5)$.
The polynomials that represent the possible length and width of the fish tank are $x + 3$ and $x + 5$.

Section 3.3 Page 135 Question 12

a) $x – 5$ is a possible factor because it is the corresponding factor for $x = 5$.
Evaluate $f(5)$.
$f(x) = x^4 − 14x^3 + 69x^2 − 140x + 100$
$f(5) = (5)^4 − 14(5)^3 + 69(5)^2 − 140(5) + 100$
$f(5) = 625 – 1750 + 1725 – 700 + 100$
$f(5) = 0$
Since $f(5) = 0$, $x – 5$ is a factor of the polynomial function.

b) Use synthetic division to find the other factors.

–5	1	–14	69	–140	100
–		–5	45	–120	100
×	1	–9	24	–20	0

Repeat the process with the remaining factor $x^3 – 9x^2 + 24x – 20$. Test factors of the constant term, –20: $P(2) = 0$.

–2	1	–9	24	–20
–		–2	14	–20
×	1	–7	10	0

Then, the remaining factor $x^2 – 7x + 10$ can be factored as $(x – 5)(x – 2)$.
So, $f(x) = (x – 5)^2(x – 2)^2$.
Since $x – 2$ is also a factor, another length of plastic that is extremely weak is a 2-ft section.

Section 3.3 Page 135 Question 13

$x^4 + 6x^3 + 11x^2 + 6x = x(x^3 + 6x^2 + 11x + 6)$
Factor $x^3 + 6x^2 + 11x + 6$ to find expressions for the other three integers.
The possible integral zeros of the polynomial are the factors of the constant term, 6: ±1, ±2, ±3, and ±6. Test these values to find a first factor: $P(–1) = 0$. Use synthetic division to find the other factors.

+1	1	6	11	6
–		1	5	6
×	1	5	6	0

Then, the remaining factor $x^2 + 5x + 6$ can be factored as $(x + 2)(x + 3)$.
So, $x^4 + 6x^3 + 11x^2 + 6x = x(x + 1)(x + 2)(x + 3)$.
Possible expressions for the other three integers are $x + 1$, $x + 2$, and $x + 3$.

Section 3.3 Page 135 Question 14

Evaluate $f(1)$.
$f(x) = ax^4 + bx^3 + cx^2 - dx + e$
$f(1) = a(1)^4 + b(1)^3 + c(1)^2 - d(1) + e$
$f(1) = a + b + c + d + e$
Since $a + b + c + d + e = 0$, this polynomial is divisible by $x - 1$.

Section 3.3 Page 135 Question 15

Let $P(x) = 2x^3 + mx^2 + nx - 3$ and $Q(x) = x^3 - 3mx^2 + 2nx + 4$.
Solve a system of equations represented by $P(2) = 0$ and $Q(2) = 0$.

$P(x) = 2x^3 + mx^2 + nx - 3$
$0 = 2(2)^3 + m(2)^2 + n(2) - 3$
$0 = 16 + 4m + 2n - 3$
$-13 = 4m + 2n$ ①

$Q(x) = x^3 - 3mx^2 + 2nx + 4$
$0 = 2^3 - 3m(2)^2 + 2n(2) + 4$
$0 = 8 - 12m + 4n + 4$
$-12 = -12m + 4n$
$-6 = -6m + 2n$ ②

Subtract equation ② from ①: $-7 = 10m$

$$m = -\frac{7}{10}$$

Substitute $m = -\frac{7}{10}$ into equation ②: $-6 = -6\left(-\frac{7}{10}\right) + 2n$

$$n = -\frac{102}{20}$$

$$n = -\frac{51}{10}$$

Section 3.3 Page 135 Question 16

a) i) For $P(x) = x^3 - 1$, the possible integral zeros of the polynomial are the factors of the constant term, –1. Test these values to find a first factor: $P(1) = 0$. Use synthetic division to find the other factors.

–1	1	0	0	–1
–		–1	–1	–1
×	1	1	1	0

Then, the remaining factor $x^2 + x + 1$ cannot be factored.
So, $P(x) = (x - 1)(x^2 + x + 1)$.

ii) For $P(x) = x^3 - 27$, the possible integral zeros of the polynomial are the factors of the constant term, –27. Test these values to find a first factor: $P(3) = 0$. Use synthetic division to find the other factors.

$$\begin{array}{r|rrrr} -3 & 1 & 0 & 0 & -27 \\ - & & -3 & -9 & -27 \\ \hline \times & 1 & 3 & 9 & 0 \end{array}$$

Then, the remaining factor $x^2 + 3x + 9$ cannot be factored.
So, $P(x) = (x - 3)(x^2 + 3x + 9)$.

iii) For $P(x) = x^3 + 1$, the possible integral zeros of the polynomial are the factors of the constant term, 1. Test these values to find a first factor: $P(-1) = 0$. Use synthetic division to find the other factors.

$$\begin{array}{r|rrrr} +1 & 1 & 0 & 0 & 1 \\ - & & 1 & -1 & 1 \\ \hline \times & 1 & -1 & 1 & 0 \end{array}$$

Then, the remaining factor $x^2 - x + 1$ cannot be factored.
So, $P(x) = (x + 1)(x^2 - x + 1)$.

iv) For $P(x) = x^3 + 64$, the possible integral zeros of the polynomial are the factors of the constant term, 64. Test these values to find a first factor: $P(-4) = 0$. Use synthetic division to find the other factors.

$$\begin{array}{r|rrrr} +4 & 1 & 0 & 0 & 64 \\ - & & 4 & -16 & 64 \\ \hline \times & 1 & -4 & 16 & 0 \end{array}$$

Then, the remaining factor $x^2 - 4x + 16$ cannot be factored.
So, $P(x) = (x + 4)(x^2 - 4x + 16)$.

b) From the results in part a), I would expect $x + y$ to be a factor of $x^3 + y^3$. Then, the remaining factor will be $x^2 - xy + y^2$.

c) From the results in part a), I would expect $x - y$ to be a factor of $x^3 - y^3$. Then, the remaining factor will be $x^2 + xy + y^2$.

d) Rewrite $x^6 + y^6$ as $(x^2)^3 + (y^2)^3$. From the results in part b),
$x^6 + y^6 = (x^2 + y^2)(x^4 - x^2y^2 + y^4)$.

Section 3.3 Page 135 Question C1

The x-intercepts of the graph of $f(x) = x^4 - 3x^2 - 4$ give the zeros of the function. In turn, these provide the binomial factors of the polynomial. Since the x-intercepts are –2 and 2, $x + 2$ and $x - 2$ are factors. Use division by one of these factors to determine the other factors.

$$\begin{array}{r|rrrrr} +2 & 1 & 0 & -3 & 0 & -4 \\ - & & 2 & -4 & 2 & -4 \\ \hline \times & 1 & -2 & 1 & -2 & 0 \end{array}$$

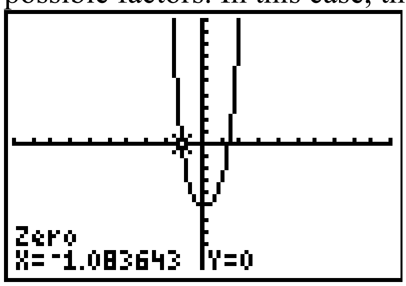
Repeat the process with $P(x) = x^3 - 2x^2 + x - 2$ with the other known factor.

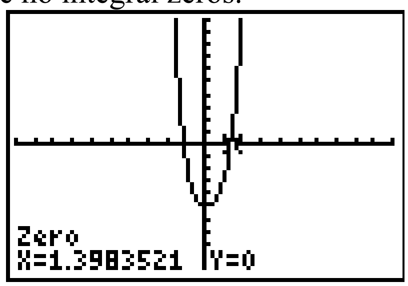
–2	1	–2	1	–2
–		–2	0	–2
×	1	0	1	0

Then, the remaining factor $x^2 + 1$ cannot be factored.
So, $f(x) = (x + 2)(x - 2)(x^2 + 1)$.

Section 3.3 Page 135 Question C2

For $P(x) = x^4 - x^3 + 2x^2 - 5$, the possible integral zeros of the polynomial are the factors of the constant term, –5: ±1 and ±5.
Graphing the polynomial and determining the x-intercepts is another method of finding possible factors. In this case, there are no integral zeros.





Section 3.3 Page 135 Question C3

You can use the factor theorem, the integral zero theorem, the quadratic formula, and synthetic division to factor a polynomial of degree greater than or equal to three.

- Use the integral zero theorem to list possible integer values for the zeros.
- Next, apply the factor theorem to determine one factor.
- Then, use synthetic division to determine the remaining factor.
- Repeat the above steps until all factors are found.
- When the remaining factor is a quadratic that cannot be factored, the exact roots may be found using the quadratic formula.

Section 3.4 Equations and Graphs of Polynomial Functions

Section 3.4 Page 147 Question 1

a) $x(x + 3)(x - 4) = 0$
$x = 0$ or $x + 3 = 0$ or $x - 4 = 0$
$x = -3$ $x = 4$

b) $(x - 3)(x - 5)(x + 1) = 0$
$x - 3 = 0$ or $x - 5 = 0$ or $x + 1 = 0$
$x = 3$ $x = 5$ $x = -1$

c) $(2x + 4)(x - 3) = 0$

$2x + 4 = 0$ or $x - 3 = 0$

$x = -2$ $x = 3$

Section 3.4 Page 147 Question 2

a) $(x + 1)^2(x + 2) = 0$

$x + 1 = 0$ or $x + 2 = 0$

$x = -1$ $x = -2$

b) $x^3 - 1 = 0$

$x^3 = 1$

$x = 1$

c) $(x + 4)^3(x + 2)^2 = 0$

$x + 4 = 0$ or $x + 2 = 0$

$x = -4$ $x = -2$

Section 3.4 Page 148 Question 3

a) Since the graph of the polynomial function crosses the x-axis at all three x-intercepts, they are of odd multiplicity. The least possible multiplicity of each x-intercept is 1, so the least possible degree is 3. The graph extends down into quadrant III and up into quadrant I, so the leading coefficient is positive. Since the x-intercepts, or roots, are –3, –2, and 1, the factors are $x + 3$, $x + 2$, and $x - 1$. The corresponding polynomial possible equation is $(x + 3)(x + 2)(x - 1) = 0$.

b) Since the graph of the polynomial function crosses the x-axis at all three x-intercepts, they are of odd multiplicity. The least possible multiplicity of each x-intercept is 1, so the least possible degree is 3. The graph extends up into quadrant II and down into quadrant IV, so the leading coefficient is negative. Since the x-intercepts, or roots, are –4, 1, and 3, the factors are $x + 4$, $x - 1$, and $x - 3$. The corresponding polynomial possible equation is $-(x + 4)(x - 1)(x - 3) = 0$.

c) Since the graph of the polynomial function crosses the x-axis at two of the x-intercepts and touches the x-axis at one of the x-intercepts, the least possible multiplicities of these x-intercepts are, respectively, 1 and 2, so the least possible degree is 4. The graph extends down into quadrant III and down into quadrant IV, so the leading coefficient is negative. Since the x-intercepts, or roots, are –4 (multiplicity 2), 1, and 3, the factors are $(x + 4)^2$, $x - 1$, and $x - 3$. The corresponding polynomial possible equation is
$-(x + 4)^2(x - 1)(x - 3) = 0$.

Section 3.4 Page 148 Question 4

a) i) The x-intercepts are -4, -1, and 1.

ii) The function is positive for values of x in the intervals $-4 < x < -1$ and $x > 1$. The function is negative for values of x in the intervals $x < -4$ and $-1 < x < 1$.

iii) Since the graph of the polynomial function crosses the x-axis at all three x-intercepts, the least possible multiplicity of each zero is 1.

b) i) The x-intercepts are -1 and 4.

ii) The function is negative for values of x in the intervals $x < -1$, $-1 < x < 4$, and $x > 4$.

iii) Since the graph of the polynomial function touches the x-axis at both x-intercepts, the least possible multiplicity of each zero is 2.

c) i) The x-intercepts are -3 and 1.

ii) The function is positive for values of x in the intervals $x < -3$ and $x > 1$. The function is negative for values of x in the interval $-3 < x < 1$.

iii) Since the graph of the polynomial function crosses the x-axis at both x-intercepts, the least possible multiplicity of each zero is 1. However, the shape of the graph close to the x-intercept of 1 is similar to the shape of the cubic curve $y = (x - 1)^3$, so that zero has multiplicity of 3.

d) i) The x-intercepts are -1 and 3.

ii) The function is positive for values of x in the interval $x < -1$. The function is negative for values of x in the intervals $-1 < x < 3$ and $x > 3$.

iii) Since the graph of the polynomial function crosses the x-axis at one of the x-intercepts and touches the x-axis at one of the x-intercepts, the least possible multiplicities of these x-intercepts are, respectively, 1 and 2. However, the shape of the graph close to the x-intercept of -1 is similar to the shape of the cubic curve $y = (x + 1)^3$, so that zero has multiplicity of 3.

Section 3.4 Page 148 Question 5

a) Compare to the graph of the base function $y = x^3$. This graph has been translated 2 units to the right and 2 units down: $h = 2$ and $k = -2$. So, the equation of the function is $y = (x - 2)^3 - 2$: choice **B**.

b) Compare to the graph of the base function $y = x^3$. This graph has been reflected in the y-axis and translated 1 unit up: $b = –1$ and $k = 1$. So, the equation of the function is $y = (–x)^3 + 1$: choice **D**.

c) Compare to the graph of the base function $y = x^4$. This graph has been stretched vertically and translated 3 units up: $a > 0$ and $k = 3$. So, the equation of the function must be $y = 0.5x^4 + 3$: choice **C**.

d) Compare to the graph of the base function $y = x^4$. This graph has been stretched horizontally and translated 1 unit to the right and 2 units down: $b > 0$, $h = 1$, and $k = –2$. So, the equation of the function must be $y = (2(x – 1))^4 – 2$: choice **A**.

Section 3.4 Page 149 Question 6

a) Compare the functions $y = 0.5(-3(x - 1))^3 + 4$ and $y = a(b(x - h))^n + k$ to determine the values of the parameters.

$b = –3$ corresponds to a horizontal stretch of factor $\frac{1}{3}$ and a reflection in the y-axis.

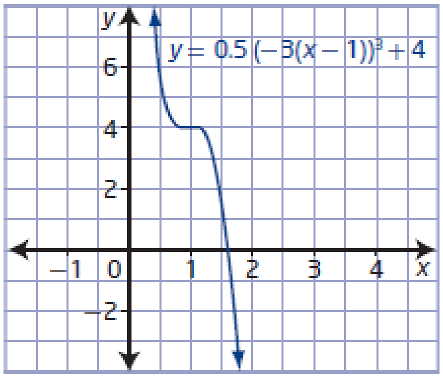
Multiply the x-coordinates of the points in column 1 by $-\frac{1}{3}$.

$a = 0.5$ corresponds to a vertical stretch of factor 0.5. Multiply the y-coordinates of the points in column 2 by 0.5.
$h = 1$ corresponds to a translation of 1 unit to the right and $k = 4$ corresponds to a translation of 4 units up. Add 1 to the x-coordinates and 4 to the y-coordinates of the points in column 3.

b)

$y = x^3$	$y = (-3x)^3$	$y = 0.5(-3x)^3$	$y = 0.5(-3(x - 1))^3 + 4$
$(-2, -8)$	$\left(\frac{2}{3}, -8\right)$	$\left(\frac{2}{3}, -4\right)$	$\left(\frac{5}{3}, 0\right)$
$(-1, -1)$	$\left(\frac{1}{3}, -1\right)$	$\left(\frac{1}{3}, -\frac{1}{2}\right)$	$\left(\frac{4}{3}, \frac{7}{2}\right)$
$(0, 0)$	$(0, 0)$	$(0, 0)$	$(1, 4)$
$(1, 1)$	$\left(-\frac{1}{3}, 1\right)$	$\left(-\frac{1}{3}, \frac{1}{2}\right)$	$\left(\frac{2}{3}, \frac{9}{2}\right)$
$(2, 8)$	$\left(-\frac{2}{3}, 8\right)$	$\left(-\frac{2}{3}, 4\right)$	$\left(\frac{1}{3}, 8\right)$

c)



Section 3.4 Page 149 Question 7

a) i) Factor the polynomial expression $x^3 - 4x^2 - 45x$.
$y = x^3 - 4x^2 - 45x$
$y = x(x^2 - 4x - 45)$
$y = x(x - 9)(x + 5)$
So, the x-intercepts of the graph are 0, 9, and –5.

ii) The function is of degree 3 and the leading coefficient is positive. The graph of the function extends down into quadrant III and up into quadrant I.

iii) The zeros are 0, 9, and –5, each of multiplicity 1.

iv) The y-intercept is 0.

v) Use the end behavior and multiplicity of the zeros. The graph begins in quadrant III, passes through $x = -5$ to above the x-axis, down through the origin to under the x-axis, and passes through $x = 9$ upward into quadrant I. The function is positive for values of x in the intervals $-5 < x < 0$ and $x > 9$. The function is negative for values of x in the intervals $x < -5$ and $0 < x < 9$.

b) i) Factor the polynomial expression $x^4 - 81x^2$.
$f(x) = x^4 - 81x^2$
$f(x) = x^2(x^2 - 81)$
$f(x) = x^2(x - 9)(x + 9)$
So, the x-intercepts of the graph are 0, 9, and –9.

ii) The function is of degree 4 and the leading coefficient is positive. The graph of the function extends up into quadrant II and up into quadrant I.

iii) The zeros are 0 (multiplicity 2) and 9 and –9, each of multiplicity 1.

iv) The y-intercept is 0.

v) Use the end behavior and multiplicity of the zeros. The graph begins in quadrant II, passes through $x = –9$ to below the x-axis, touches the x-axis at $x = 0$, passes through $x = 9$ upward into quadrant I. The function is positive for values of x in the intervals $x < –9$ and $x > 9$. The function is negative for values of x in the intervals $–9 < x < 0$ and $0 < x < 9$.

c) **i)** Factor the polynomial expression $x^3 + 3x^2 - x - 3$.
$h(x) = x^3 + 3x^2 - x - 3$
$h(x) = x^2(x + 3) – (x + 3)$
$h(x) = (x + 3)(x^2 – 1)$
$h(x) = (x + 3)(x – 1)(x + 1)$

So, the x-intercepts of the graph are –3, 1, and –1.

ii) The function is of degree 3 and the leading coefficient is positive. The graph of the function extends down into quadrant III and up into quadrant I.

iii) The zeros are –3, 1, and –1, each of multiplicity 1.

iv) The y-intercept is –3.

v) Use the end behavior and multiplicity of the zeros. The graph begins in quadrant III, passes through $x = –3$ to above the x-axis, passes through $x = –1$ to below the x-axis, passes through the y-intercept and the x-axis at $x = 1$ upward into quadrant I. The function is positive for values of x in the intervals $–3 < x < –1$ and $x > 1$. The function is negative for values of x in the intervals $x < –3$ and $–1 < x < 1$.

d) **i)** For $k(x) = –x^4 – 2x^3 + 7x^2 + 8x - 12$, the possible integral zeros of the polynomial are the factors of the constant term, –12: ±1, ±2, ±3, ±4, ±6, and ±12. Test these values to find a first factor: $k(1) = 0$. Use synthetic division to find the other factors.

–1	–1	–2	7	8	–12
–		1	3	–4	–12
×	–1	–3	4	12	0

The remaining factor $–x^3 – 3x^2 + 4x + 12$ can be factored.
$–x^3 – 3x^2 + 4x + 12$
$= –x^2(x + 3) + 4(x + 3)$
$= (x + 3)(–x^2 + 4)$
$= –(x + 3)(x^2 – 4)$
$= –(x + 3)(x + 2)(x – 2)$
So, $k(x) = –(x – 1)(x + 3)(x + 2)(x – 2)$.
The x-intercepts of the graph are 1, –3, –2, and 2.

ii) The function is of degree 4 and the leading coefficient is negative. The graph of the function extends down into quadrant III and down into quadrant IV.

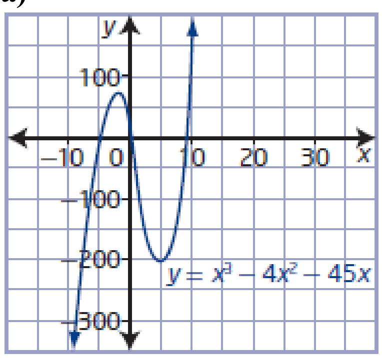
iii) The zeros are 1, –3, –2, and 2, each of multiplicity 1.

iv) The *y*-intercept is –12.

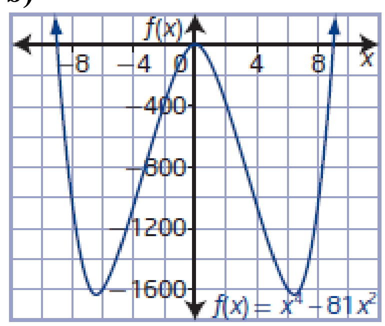
v) Use the end behavior and multiplicity of the zeros. The graph begins in quadrant III, passes through $x = -3$ to above the *x*-axis, passes down through $x = -2$ to below the *x*-axis, passes through the *y*-intercept and up through the $x = 1$, and back down through $x = 2$ into quadrant IV. The function is positive for values of *x* in the intervals $-3 < x < -2$ and $1 < x < 2$. The function is negative for values of *x* in the intervals $x < -3$, $-2 < x < 1$, and $x > 2$.

Section 3.4 Page 149 Question 8

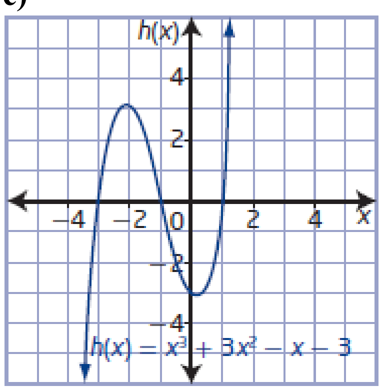
a)



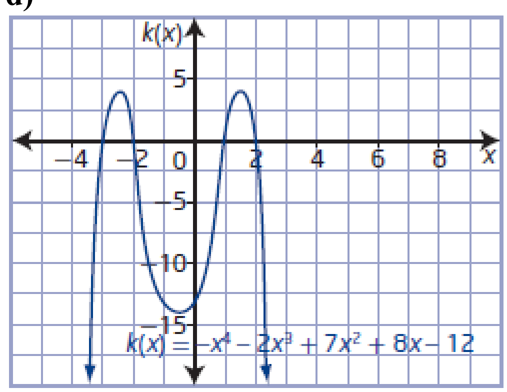
b)



c)



d)



Section 3.4 Page 149 Question 9

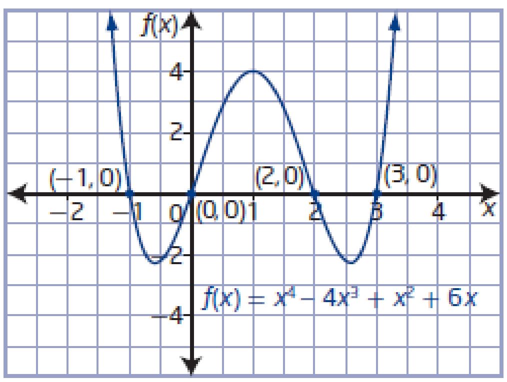
a) First factor out the common factor.
$f(x) = x^4 - 4x^3 + x^2 + 6x$
$f(x) = x(x^3 - 4x^2 + x + 6)$
For $P(x) = x^3 - 4x^2 + x + 6$, the possible integral zeros of the polynomial are the factors of the constant term, 6: ±1, ±2, ±3, and ±6. Test these values to find a first factor: $P(-1) = 0$. Use synthetic division to find the other factors.

+1	1	–4	1	6
–		1	–5	6
×	1	–5	6	0

Then, the remaining factor $x^2 - 5x + 6$ can be factored as $(x - 2)(x - 3)$.
So, $f(x) = x(x + 1)(x - 2)(x - 3)$.

Use a table to organize information about the function. Then, use the information to sketch the graph.

Degree	4
Leading Coefficient	1
End Behaviour	extends up into quadrant II and up into quadrant I
Zeros/*x*-Intercepts	−1, 0, 2, and 3
***y*-Intercept**	0
Intervals Where the Function Is Positive or Negative	positive values of $f(x)$ in the intervals $x < -1$, $0 < x < 2$, and $x > 3$ negative values of $f(x)$ in the intervals $-1 < x < 0$ and $2 < x < 3$



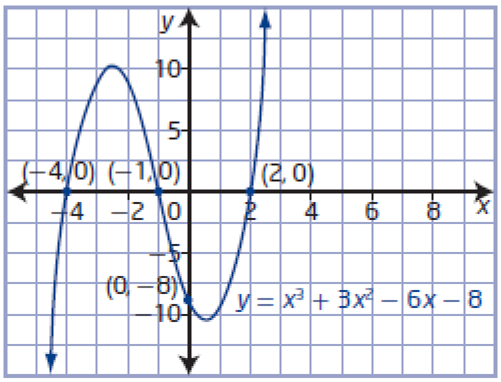
b) For $P(x) = x^3 + 3x^2 - 6x - 8$, the possible integral zeros of the polynomial are the factors of the constant term, –8: ±1, ±2, ±4, and ±8. Test these values to find a first factor: $P(-1) = 0$. Use synthetic division to find the other factors.

+1	1	3	–6	–8
–		1	2	–8
×	1	2	–8	0

Then, the remaining factor $x^2 + 2x - 8$ can be factored as $(x + 4)(x - 2)$.
So, $y = (x + 1)(x + 4)(x - 2)$.

Use a table to organize information about the function. Then, use the information to sketch the graph.

Degree	3
Leading Coefficient	1
End Behaviour	extends down into quadrant III and up into quadrant I
Zeros/*x*-Intercepts	−4, –1, and 2
***y*-Intercept**	–8
Intervals Where the Function Is Positive or Negative	positive values of $f(x)$ in the intervals $-4 < x < -1$ and $x > 2$ negative values of $f(x)$ in the intervals $x < -4$ and $-1 < x < 2$



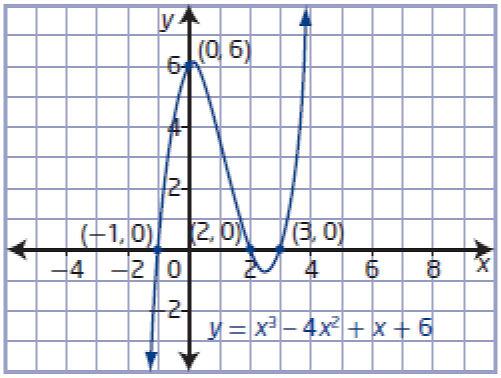
c) For $P(x) = x^3 - 4x^2 + x + 6$, the possible integral zeros of the polynomial are the factors of the constant term, 6: ±1, ±2, ±3, and ±6. Test these values to find a first factor: $P(-1) = 0$. Use synthetic division to find the other factors.

+1	1	–4	1	6
–		1	–5	6
×	1	–5	6	0

Then, the remaining factor $x^2 - 5x + 6$ can be factored as $(x - 2)(x - 3)$.
So, $y = (x + 1)(x - 2)(x - 3)$.

Use a table to organize information about the function. Then, use the information to sketch the graph.

Degree	3
Leading Coefficient	1
End Behaviour	extends down into quadrant III and up into quadrant I
Zeros/*x*-Intercepts	−1, 2, and 3
***y*-Intercept**	6
Intervals Where the Function Is Positive or Negative	positive values of $f(x)$ in the intervals $-1 < x < 2$ and $x > 3$ negative values of $f(x)$ in the intervals $x < -1$ and $2 < x < 3$



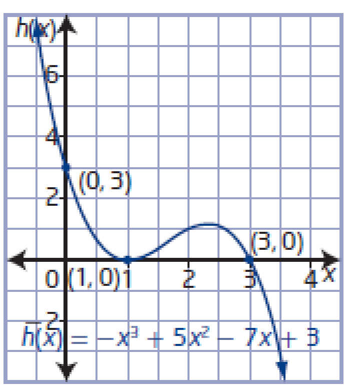
d) For $P(x) = -x^3 + 5x^2 - 7x + 3$, the possible integral zeros of the polynomial are the factors of the constant term, 3: ±1 and ±3. Test the values to find a first factor: $P(1) = 0$. Use synthetic division to find the other factors.

$$\begin{array}{r|rrrr} -1 & -1 & 5 & -7 & 3 \\ - & & 1 & -4 & 3 \\ \hline \times & -1 & 4 & -3 & 0 \end{array}$$

Then, the remaining factor $-x^2 + 4x - 3$ can be factored as $-(x - 1)(x - 3)$.
So, $h(x) = -(x - 1)^2(x - 3)$.

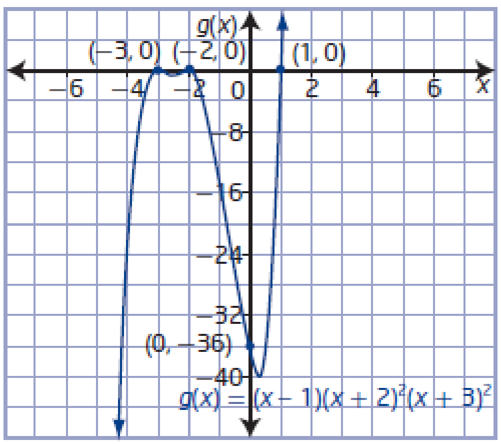
Use a table to organize information about the function. Then, use the information to sketch the graph.

Degree	3
Leading Coefficient	–1
End Behaviour	extends up into quadrant II and down into quadrant IV
Zeros/*x*-Intercepts	1 (multiplicity 2) and 3
***y*-Intercept**	3
Intervals Where the Function Is Positive or Negative	positive values of $f(x)$ in the intervals $x < 1$ and $1 < x < 3$ negative values of $f(x)$ in the interval $x > 3$



e) The function $g(x) = (x - 1)(x + 2)^2(x + 3)^2$ is in factored form.
Use a table to organize information about the function. Then, use the information to sketch the graph.

Degree	When the function is expanded, the exponent of the highest-degree term is 5. The function is of degree 5.
Leading Coefficient	When the function is expanded, the leading coefficient is $1(1^2)(1^2)$ or 1.
End Behaviour	extends down into quadrant III and up into quadrant I
Zeros/*x*-Intercepts	–3 (multiplicity 2), –2 (multiplicity 2), and 1
***y*-Intercept**	$(0 - 1)(0 + 2)^2(0 + 3)^2 = -36$
Intervals Where the Function Is Positive or Negative	positive values of $f(x)$ in the interval $x > 1$ negative values of $f(x)$ in the intervals $x < -3$, $-3 < x < -2$, and $-2 < x < 1$



f) For $P(x) = -x^4 - 2x^3 + 3x^2 + 4x - 4$, the possible integral zeros of the polynomial are the factors of the constant term, –4: ±1, ±2, and ±4. Test these values to find a first factor: $P(1) = 0$. Use synthetic division to find the other factors.

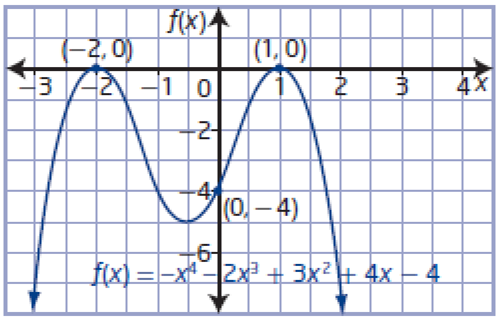
$$\begin{array}{r|rrrrr} -1 & -1 & -2 & 3 & 4 & -4 \\ - & & 1 & 3 & 0 & -4 \\ \hline \times & -1 & -3 & 0 & 4 & 0 \end{array}$$

Repeat the process with the remaining factor $-x^3 - 3x^2 + 4$. Test factors of the constant term, 4: $P(1) = 0$.

$$\begin{array}{r|rrrr} -1 & -1 & -3 & 0 & 4 \\ - & & 1 & 4 & 4 \\ \hline \times & -1 & -4 & -4 & 0 \end{array}$$

Then, the remaining factor $-x^2 - 4x - 4$ can be factored as $-(x + 2)^2$.
So, $f(x) = -(x - 1)^2(x + 2)^2$.
Use a table to organize information about the function. Then, use the information to sketch the graph.

Degree	4
Leading Coefficient	–1
End Behaviour	extends down into quadrant III and down into quadrant IV
Zeros/*x*-Intercepts	–2 (multiplicity 2) and 1 (multiplicity 2)
***y*-Intercept**	–4
Intervals Where the Function Is Positive or Negative	positive values of $f(x)$ in no interval negative values of $f(x)$ in the intervals $x < -2$, $-2 < x < 1$, and $x > 1$



Section 3.4 Page 149 Question 10

a) Since the graph extends down into quadrant III and up into quadrant I, the sign of the leading coefficient is positive. The x-intercepts are –2 (multiplicity 3) and 3 (multiplicity 2). The function is positive for values of x in the intervals $-2 < x < 3$ and $x > 3$. The function is negative for values of x in the interval $x < -2$. The equation for the polynomial function is $y = (x + 2)^3(x - 3)^2$.

b) Since the graph extends up into quadrant II and down into quadrant IV, the sign of the leading coefficient is negative. The x-intercepts are –4, –1, and 3. The function is positive for values of x in the intervals $x < -4$ and $-1 < x < 3$. The function is negative for values of x in the intervals $-4 < x < -1$ and $x > 3$. The equation for the polynomial function is $y = -(x + 4)(x + 1)(x - 3)$.

c) Since the graph extends down into quadrant III and down into quadrant IV, the sign of the leading coefficient is negative. The x-intercepts are –2, –1, 2, and 3. The function is positive for values of x in the intervals $-2 < x < -1$ and $2 < x < 3$. The function is negative for values of x in the intervals $x < -2$, $-1 < x < 2$, and $x > 3$. The equation for the polynomial function is $y = -(x + 2)(x + 1)(x - 2)(x - 3)$.

d) Since the graph extends up into quadrant II and up into quadrant I, the sign of the leading coefficient is positive. The x-intercepts are –1, 1, and 3 (multiplicity 2). The function is positive for values of x in the intervals $x < -1$, $1 < x < 3$, and $x > 3$. The function is negative for values of x in the interval $-1 < x < 1$. The equation for the polynomial function is $y = (x + 1)(x - 1)(x - 3)^2$.

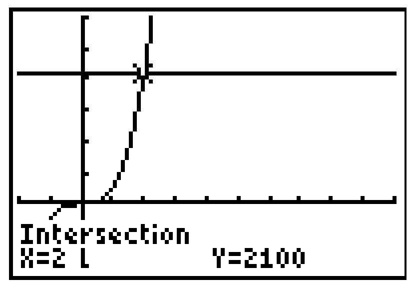
Section 3.4 Page 150 Question 11

a) For $y = \left(\frac{1}{2}(x-2)\right)^3 - 3$, $a = 1$, $b = \frac{1}{2}$, $h = 2$, and $k = -3$.

b) $a = 1$: no vertical stretch
$b = \frac{1}{2}$: horizontal stretch by a factor of 2
$h = 2$: horizontal translation of 2 units to the right
$k = -3$: vertical translation of 3 units down

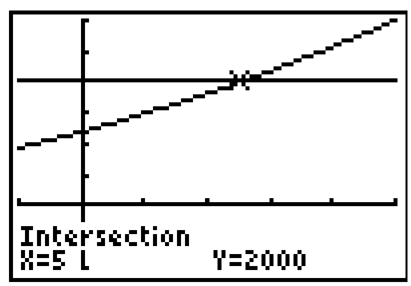
c) The domain is $\{x \mid x \in \mathrm{R}\}$ and the range is $\{y \mid y \in \mathrm{R}\}$.

Section 3.4 Page 150 Question 12



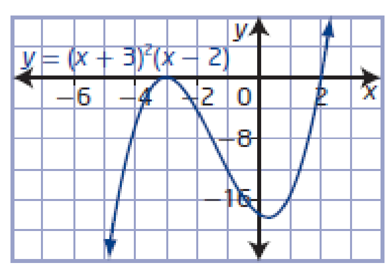
Solve $2100 = x(25x)(10x + 1)$ by graphing.
The x-coordinate of the point of intersection is 2.
So, the dimensions of the pool are 2 m by 50 m by 21 m.

Section 3.4 Page 150 Question 13



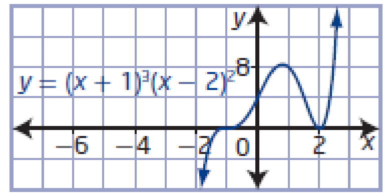
The combined surface area of the pond and the boardwalk are given by $SA = (30 + 2x)(40 + 2x)$.
Solve $2000 = (30 + 2x)(40 + 2x)$ graphically.
At the point of intersection, $x = 5$.
The width of the boardwalk is 5 ft.

Section 3.4 Page 150 Question 14



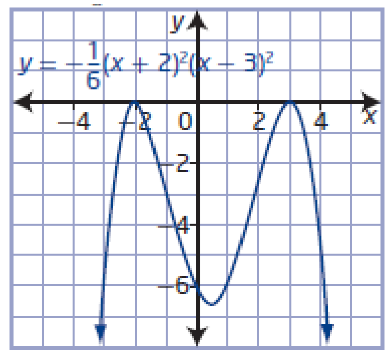
a) For a cubic function with zeros –3 (multiplicity 2) and 2 and y-intercept –18, the corresponding factors are $(x + 3)^2$ and $(x – 2)$ and $a_0 = –18$.
Try an equation with just the factors: $y = (x + 3)^2(x – 2)$.
Check the y-intercept.
$y = (0 + 3)^2(0 – 2)$
$y = –18$
The equation with least degree is $y = (x + 3)^2(x – 2)$.

b) For a quintic function with zeros –1 (multiplicity 3) and 2 (multiplicity 2) and y-intercept 4, the corresponding factors are $(x + 1)^3$ and $(x – 2)^2$ and $a_0 = 4$.
Try an equation with just the factors: $y = (x + 1)^3(x – 2)^2$.
Check the y-intercept.
$y = (0 + 1)^3(0 – 2)^2$
$y = 4$
The equation with least degree is $y = (x + 1)^3(x – 2)^2$.



c) For a quartic function with a negative leading coefficient, zeros –2 (multiplicity 2) and 3 (multiplicity 2) and a constant term of –6, the corresponding factors are $(x + 2)^2$ and $(x – 3)^2$ and $a_0 = –6$.
Try an equation with just the factors: $y = –(x + 2)^2(x – 3)^2$.
Check the constant term.
$y = –(0 + 2)^2(0 – 3)^2$
$y = –36$
To adjust the constant term, use $a = \frac{1}{6}$ then $a_0 = –6$.

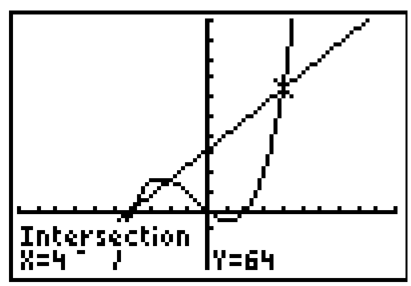
The equation with least degree is $y = -\frac{1}{6}(x + 2)^2(x – 3)^2$.



Section 3.4 Page 150 Question 15

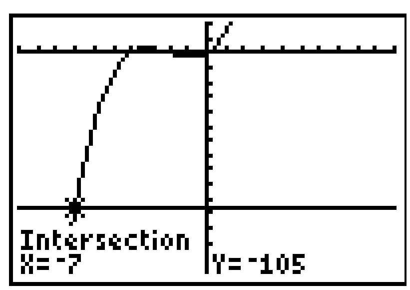
The width of a rectangular prism is w. Then, the height is $w – 2$, the length is $w + 4$, and $V = 8(w + 4)$.
Solve $8(w + 4) = w(w – 2)(w + 4)$ graphically.

The dimensions of the prism are 4 cm by 2 cm by 8 cm.



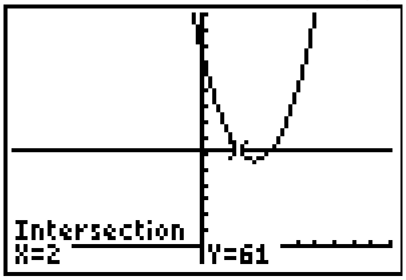
Section 3.4 Page 150 Question 16

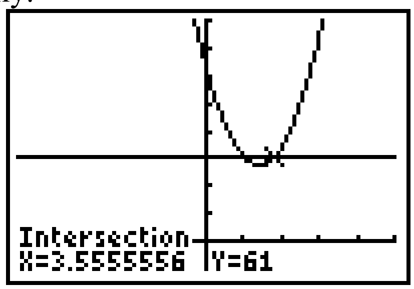
Let three consecutive odd numbers be represented by x, $x + 2$, and $x + 4$.
Solve $–105 = x(x + 2)(x + 4)$ graphically.
The three integers are –7, –5, and –3.



Section 3.4 Page 150 Question 17

Let x represent the side length of the smaller cube. Then $5 - x$ represents the side length of the larger cube.
Determine an expression for the exposed surface area.
$SA_E = SA_S + SA_L$
$SA_E = 5x^2 + 5(5 - x)^2 - x^2$
$SA_E = 4x^2 + 125 - 50x + 5x^2$
$SA_E = 9x^2 - 50x + 125$
Solve $61 = 9x^2 - 50x + 125$ graphically.



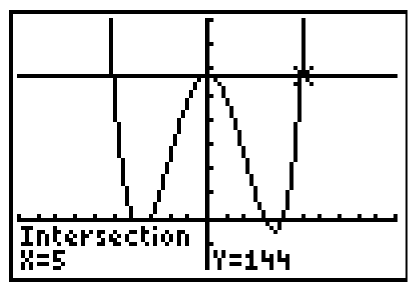


Since x represents the smaller cube, $x = 2$ is the only acceptable answer.
The side length of the smaller cube is 2 m and the side length of the larger cube is 3 m.

Section 3.4 Page 151 Question 18

a) The area of the border can be modelled by the polynomial expression $(x^2 - 12)^2 - x^2$.

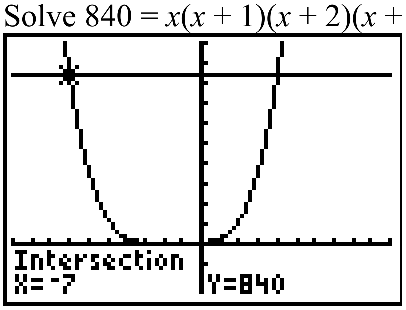
b) Solve $144 = (x^2 - 12)^2 - x^2$ graphically.
The dimensions of Olutie's wall hanging are 5 in. by 5 in.

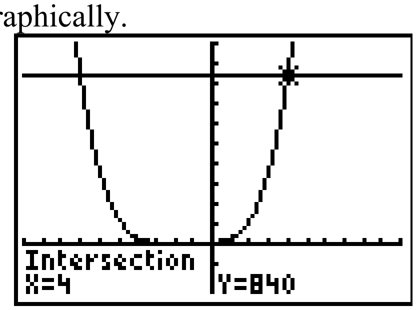


c) The dimensions of the border are 13 in. by 13 in.

Section 3.4 Page 151 Question 19

Let four consecutive integers be represented by x, $x + 1$, $x + 2$, and $x + 3$.
Solve $840 = x(x + 1)(x + 2)(x + 3)$ graphically.





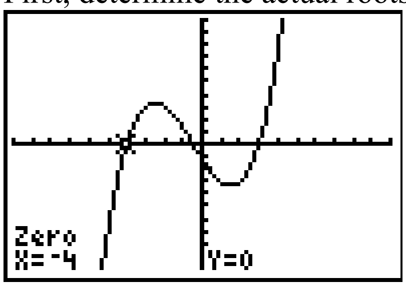
The four consecutive integers are –7, –6, –5, and –4 or 4, 5, 6, and 7.

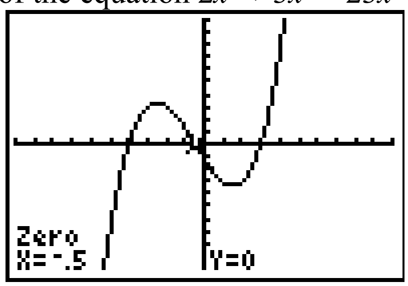
Section 3.4 Page 151 Question 20

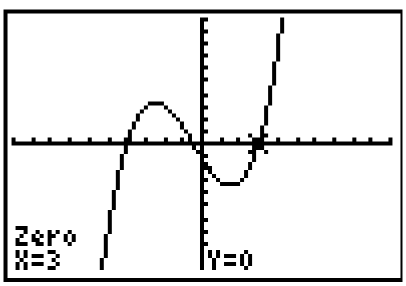
For a cubic function with x-intercepts of $\sqrt{3}$, $-\sqrt{3}$, and 1 and a y-intercept of -1, the corresponding factors are $x - \sqrt{3}$, $x + \sqrt{3}$, and $x - 1$ and $a_0 = -1$.
Try an equation with just the factors: $y = (x - \sqrt{3})(x + \sqrt{3})(x - 1)$.
Check the y-intercept.
$y = (0 - \sqrt{3})(0 + \sqrt{3})(0 - 1)$
$y = 3$
The equation of the cubic function is $y = -\frac{1}{3}(x - \sqrt{3})(x + \sqrt{3})(x - 1)$.

Section 3.4 Page 151 Question 21

First, determine the actual roots of the equation $2x^3 + 3x^2 - 23x - 12 = 0$.







So, $a = -4$, $b = -0.5$, and $c = 3$.

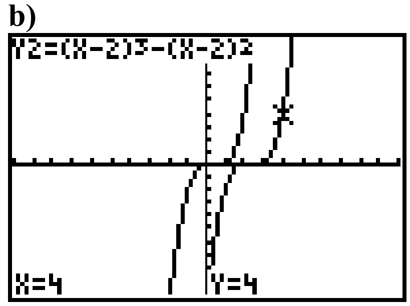
Then, $a + b = -4.5$, $\frac{a}{b} = 8$, and $ab = 2$.

An equation with these roots is $(x + 4.5)(x - 8)(x - 2) = 0$.

Section 3.4 Page 151 Question 22

a) I predict that the graph of $y = (x - 2)^3 - (x - 2)^2$ represents a horizontal translation by 2 units to the right of the graph of $y = x^3 - x^2$.

b)

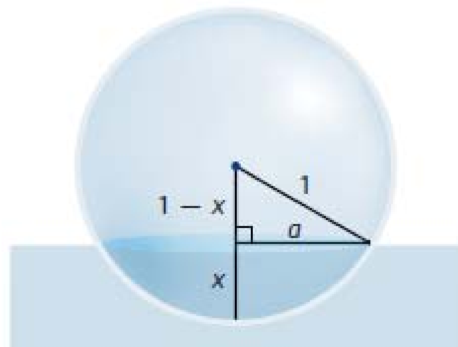


c) $y = x^3 - x^2$
$y = x^2(x - 1)$
$0 = x^2(x - 1)$
$x^2 = 0$ or $x - 1 = 0$
$x = 0$ $x = 1$
The x-intercepts are 0 and 1.

$y = (x - 2)^3 - (x - 2)^2$
$y = (x - 2)^2(x - 2 - 1)$
$y = (x - 2)^2(x - 3)$
$0 = (x - 2)^2(x - 3)$
$(x - 2)^2 = 0$ or $x - 3 = 0$
$x = 2$ $x = 3$
The x-intercepts are 2 and 3.

Section 3.4 Page 151 Question 23

Use the information in the diagram and the Pythagorean theorem to determine an expression for a^2 in terms of x: $a^2 = 2x - x^2$. Substitute the expression for a^2 into $V_{\text{cap}} = \frac{\pi x}{6}(3a^2 + x^2)$.



$$V_{\text{cap}} = \frac{\pi x}{6}(3a^2 + x^2)$$

$$V_{\text{cap}} = \frac{\pi x}{6}(3(2x - x^2) + x^2)$$

$$V_{\text{cap}} = \frac{\pi x}{6}(6x - 2x^2)$$

$$V_{\text{cap}} = \frac{\pi x^2}{3}(3 - x)$$

$$V_{\text{cap}} = V_{\text{disp}} = \frac{\pi x^2}{3}(3 - x)$$

Then, substitute $V_{\text{disp}} = \frac{\pi x^2}{3}(3 - x)$, $\rho_{\text{buoy}} = \frac{1}{4}\rho_{\text{fluid}}$, where ρ represents density, and $V_{\text{buoy}} = \frac{4}{3}\pi(1)^2$ into $\rho_{\text{fluid}}V_{\text{disp}} = \rho_{\text{buoy}}V_{\text{buoy}}$ and solve for x.

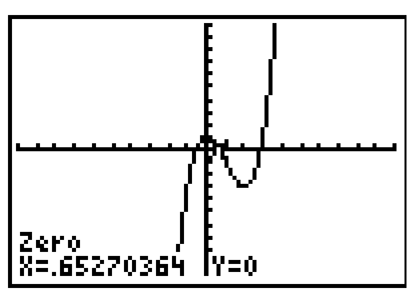
$$\rho_{\text{fluid}}V_{\text{disp}} = \rho_{\text{buoy}}V_{\text{buoy}}$$

$$\rho_{\text{fluid}}\left(\frac{\pi x^2}{3}(3 - x)\right) = \frac{1}{4}\rho_{\text{fluid}}\left(\frac{4\pi}{3}(1)^2\right)$$

$$\pi x^2(3 - x) = \pi$$

$$3x^2 - x^3 = 1$$

$$x^3 - 3x^2 + 1 = 0$$



Graph the corresponding function and determine the value of the zero that makes sense in this case. So, $x \approx 0.65$.

The buoy sinks to a depth of approximately 0.65 m.

Section 3.4 Page 152 Question C1

Example: It is easier to identify the zeros when a polynomial is in factored form. For example,
$y = (x - 1)(x + 2)(x + 4)$ instead of $y = x^3 + 5x^2 + 2x - 8$.

Section 3.4 Page 152 Question C2

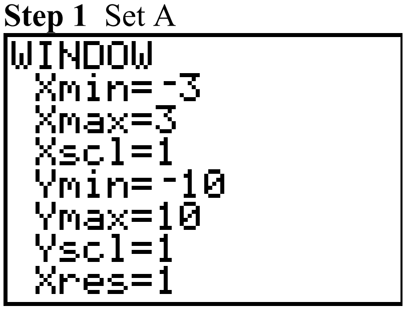
A root of an equation is a solution of the equation. A zero of a function is a value of x for which $f(x) = 0$. An x-intercept of a graph is the x-coordinate of the point where a line or curve crosses or touches the x-axis. They all represent the same value for x.

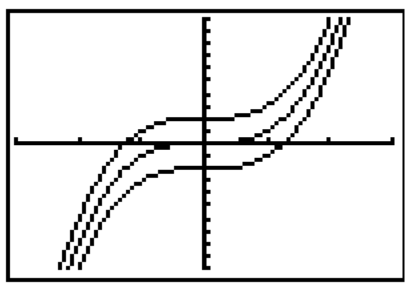
Section 3.4 Page 152 Question C3

Example: If the multiplicity of a zero is 1, the graph crosses the x-axis at that value. If the multiplicity of a zero is even, the graph only touches the x-axis at that value. The shape of a graph close to a zero of $x = a$ (multiplicity n, where n is an odd number greater than 1) is similar to the shape of the graph of a function with degree equal to n of the form $y = (x - a)^n$.

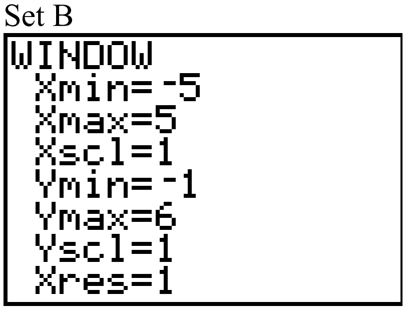
Section 3.4 Page 152 Question C4

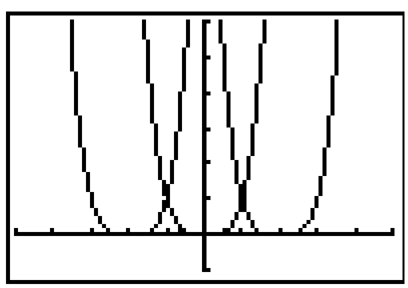
Step 1 Set A





Set B





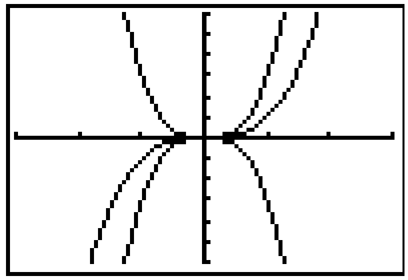
a) The graph of $y = x^3 + k$ is translated vertically k units compared to the graph of $y = x^3$. For example, $y = x^3 + 3$ is translated vertically 3 units compared to the graph of $y = x^3$.

b) The graph of $y = (x - h)^4$ is translated horizontally h units compared to the graph of $y = x^4$. For example, $y = (x + 2)^4$ is translated horizontally 2 units to the left compared to the graph of $y = x^4$.

Step 2 In functions of the form $y = a(b(x - h))^n + k$, parameter h represents any horizontal translation while parameter k represents any vertical translation.

Step 3 Set C

WINDOW
Xmin=-3
Xmax=3
Xscl=1
Ymin=-6
Ymax=6
Yscl=1
Xres=1



Set D

WINDOW
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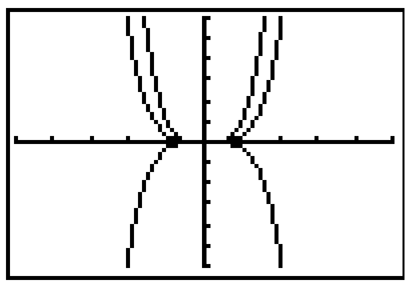
Xmax=5
Xscl=1
Ymin=-6
Ymax=6
Yscl=1
Xres=1



a) The graph of $y = ax^3$ is stretched vertically by a factor of $|a|$ relative to the graph of $y = x^3$. When a is negative, the graph is reflected in the x-axis.

b) When the value of a is $-1 < a < 0$ or $0 < a < 1$, the graph of $y = ax^4$ is stretched vertically by a factor of $|a|$ relative to the graph of $y = x^4$. When a is negative, the graph is reflected in the x-axis.

Step 4 Set E

WINDOW
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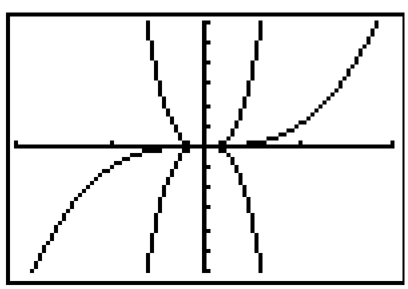
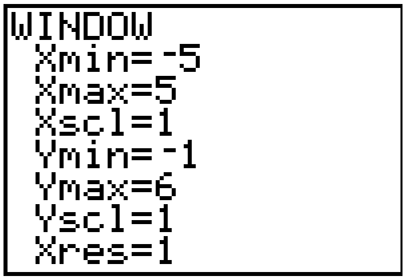
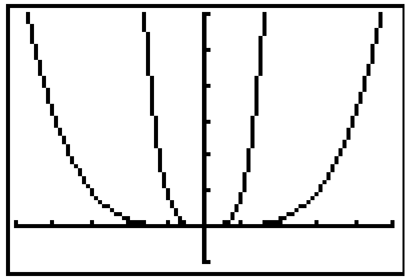
Xmax=2
Xscl=1
Ymin=-6
Ymax=6
Yscl=1
Xres=1



Set F





a) The graph of $y = (bx)^3$ is stretched horizontally by a factor of $\frac{1}{|b|}$ relative to the graph of $y = x^3$. When b is negative, the graph is reflected in the y-axis.

b) When the value of b is $-1 < b < 0$ or $0 < b < 1$, the graph of $y = (bx)^4$ is stretched horizontally by a factor of $\frac{1}{|b|}$ relative to the graph of $y = x^4$. When b is negative, the graph is reflected in the y-axis.

Step 5 In functions of the form $y = a(b(x - h))^n + k$, parameter a represents any vertical stretch and/or reflection in the x-axis while parameter b represents any horizontal stretch and/or reflection in the y-axis.

Chapter 3 Review

Chapter 3 Review Page 153 Question 1

a) The function $y = \sqrt{x+1}$ is a radical function, not a polynomial function.
$\sqrt{x+1}$ is the same as $(x+1)^{\frac{1}{2}}$, which has an exponent that is not a whole number.

b) The function $f(x) = 3x^4$ is of the form $g(x) = a_4x^4 + a_3x^3 + a_2x^2 + a_1x + a_0$.
It is a polynomial of degree 4. The leading coefficient is 3 and the constant term is 0.

c) The function $g(x) = –3x^3 – 2x^2 + x$ is of the form $g(x) = a_3x^3 + a_2x^2 + a_1x + a_0$.
It is a polynomial of degree 3. The leading coefficient is –3 and the constant term is 0.

d) The function $y = \frac{1}{2}x + 7$ is of the form $g(x) = a_1x + a_0$.

It is a polynomial of degree 1. The leading coefficient is $\frac{1}{2}$ and the constant term is 7.

Chapter 3 Review Page 153 Question 2

a) The function $s(x) = x^4 - 3x^2 + 5x$ is a quartic (degree 4), which is an even-degree polynomial function. Its graph has a maximum of four x-intercepts. Since the leading coefficient is positive, the graph of the function opens upward, extending up into quadrant II and up into quadrant I, and has a minimum value. The graph has a y-intercept of 0.

b) The function $p(x) = -x^3 + 5x^2 - x + 4$ is a cubic (degree 3), which is an odd-degree polynomial function. Its graph has at least one x-intercept and at most three x-intercepts. Since the leading coefficient is negative, the graph of the function extends up into quadrant II and down into quadrant IV. The graph has no maximum or minimum values. The graph has a y-intercept of 4.

c) The function $y = 3x - 2$ is linear (degree 1), which is an odd-degree polynomial function. Its graph has one x-intercept. Since the leading coefficient is positive, the graph of the function extends down into quadrant III and up into quadrant I, and has a no maximum or minimum value. The graph has a y-intercept of -2.

d) The function $y = 2x^2 - 4$ is a quadratic (degree 2), which is an even-degree polynomial function. Its graph has a maximum of two x-intercepts. Since the leading coefficient is positive, the graph of the function opens upward, extending up into quadrant II and up into quadrant I, and has a minimum value. The graph has a y-intercept of -4.

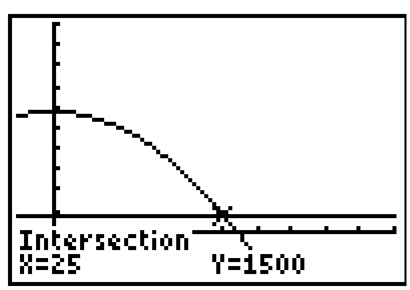
e) The function $y = 2x^5 - 3x^3 + 1$ is a quintic (degree 5), which is an odd-degree polynomial function. Its graph has at least one x-intercept and at most five x-intercepts. Since the leading coefficient is positive, the graph of the function extends down into quadrant III and up into quadrant I. The graph has no maximum or minimum values. The graph has a y-intercept of 1.

Chapter 3 Review Page 153 Question 3

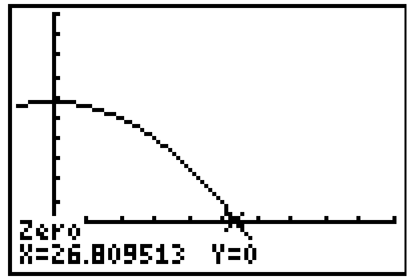
a) The function $h(t) = 11\,500 - 16t^2$ is a quadratic function (degree 2).

b) Substitute $t = 12$.
$h(12) = 11\,500 - 16(12)^2$
$h(12) = 9196$
The parachutist's height above the ground after 12 s will be 9196 ft.

c) Solve $1500 = 11\,500 - 16t^2$ graphically.
The parachutist will be 1500 ft above the ground after 25 s.



d) Solve $0 = 11\ 500 - 16t^2$ graphically.
The parachutist will reach the ground after approximately 26.81 s.



Chapter 3 Review Page 153 Question 4

a) $P(x) = x^3 + 9x^2 - 5x + 3$
$P(2) = 2^3 + 9(2)^2 - 5(2) + 3$
$P(2) = 8 + 36 - 10 + 3$
$P(2) = 37$
The remainder when $x^3 + 9x^2 - 5x + 3$ is divided by $x - 2$ is 37.

For $(x^3 + 9x^2 - 5x + 3) \div (x - 2)$, $x \neq 2$.

$$\begin{array}{r}
x^2 + 11x + 17 \\
x-2 \overline{\big) x^3 + 9x^2 - 5x + 3} \\
\underline{x^3 - 2x^2} \qquad\qquad \\
11x^2 - 5x \qquad \\
\underline{11x^2 - 22x} \qquad \\
17x + 3 \\
\underline{17x - 34} \\
37
\end{array}$$

$$\frac{x^3 + 9x^2 - 5x + 3}{x - 2} = x^2 + 11x + 17 + \frac{37}{x - 2}$$

b) $P(x) = 2x^3 + x^2 - 2x + 1$
$P(-1) = 2(-1)^3 + (-1)^2 - 2(-1) + 1$
$P(-1) = -2 + 1 + 2 + 1$
$P(-1) = 2$
The remainder when $2x^3 + x^2 - 2x + 1$ is divided by $x + 1$ is 2.

For $(2x^3 + x^2 - 2x + 1) \div (x + 1)$, $x \neq -1$.

$$
\begin{array}{r l}
 & \quad\;\; 2x^2 - x - 1 \\
x+1 & \overline{)\,2x^3 + x^2 - 2x + 1} \\
 & \underline{\;2x^3 + 2x^2} \\
 & \qquad -x^2 - 2x \\
 & \qquad \underline{-x^2 - x} \\
 & \qquad\qquad\quad -x + 1 \\
 & \qquad\qquad\quad \underline{-x - 1} \\
 & \qquad\qquad\qquad\quad 2
\end{array}
$$

$$\frac{2x^3 + x^2 - 2x + 1}{x+1} = 2x^2 - x - 1 + \frac{2}{x+1}$$

c) $P(x) = 12x^3 + 13x^2 - 23x + 7$
$P(1) = 12(1)^3 + 13(1)^2 - 23(1) + 7$
$P(1) = 12 + 13 - 23 + 7$
$P(1) = 9$
The remainder when $12x^3 + 13x^2 - 23x + 7$ is divided by $x - 1$ is 9.

For $(12x^3 + 13x^2 - 23x + 7) \div (x - 1)$, $x \neq 1$.

–1	12	13	–23	7
–		–12	–25	–2
×	12	25	2	9

$$\frac{12x^3 + 13x^2 - 23x + 7}{x-1} = 12x^2 + 25x + 2 + \frac{9}{x-1}$$

d) Ensure the polynomial is written in descending powers of x: $-8x^4 + 10x^3 - 4x + 15$.
$P(x) = -8x^4 + 10x^3 - 4x + 15$
$P(-1) = -8(-1)^4 + 10(-1)^3 - 4(-1) + 15$
$P(-1) = -8 - 10 + 4 + 15$
$P(-1) = 1$
The remainder when $-8x^4 + 10x^3 - 4x + 15$ is divided by $x + 1$ is 1.

For $(-8x^4 + 10x^3 - 4x + 15) \div (x + 1)$, $x \neq -1$.

+1	–8	10	0	–4	15
–		–8	18	–18	14
×	–8	18	–18	14	1

$$\frac{-8x^4 - 4x + 10x^3 + 15}{x+1} = -8x^3 + 18x^2 - 18x + 14 + \frac{1}{x+1}$$

Chapter 3 Review Page 153 Question 5

a) Solve $P(3) = -14$ to determine the value of k.

$f(x) = x^4 + kx^3 - 3x - 5$
$-14 = 3^4 + k(3)^3 - 3(3) - 5$
$-14 = 81 + 27k - 9 - 5$
$-81 = 27k$
$k = -3$

b) Evaluate $f(-3)$.

$f(x) = x^4 - 3x^3 - 3x - 5$
$f(-3) = (-3)^4 - 3(-3)^3 - 3(-3) - 5$
$f(-3) = 81 + 81 + 9 - 5$
$f(-3) = 166$

The remainder when $x^4 + kx^3 - 3x - 5$ is divided by $x + 3$ is 166.

Chapter 3 Review Page 153 Question 6

First, determine an expression for $P(1)$ and $P(-3)$.

$P(x) = 4x^3 - 3x^2 + bx + 6$
$P(1) = 4(1)^3 - 3(1)^2 + b(1) + 6$
$P(1) = 4 - 3 + b + 6$
$P(1) = b + 7$

$P(x) = 4x^3 - 3x^2 + bx + 6$
$P(-3) = 4(-3)^3 - 3(-3)^2 + b(-3) + 6$
$P(-3) = -108 - 27 - 3b + 6$
$P(-3) = -3b - 129$

Then, solve $P(1) = P(-3)$ to determine the value of b.

$b + 7 = -3b - 129$
$4b = -136$
$b = -34$

Chapter 3 Review Page 153 Question 7

a) Evaluate $P(1)$.

$P(x) = x^3 - x^2 - 16x + 16$
$P(1) = 1^3 - 1^2 - 16(1) + 16$
$P(1) = 1 - 1 - 16 + 16$
$P(1) = 0$

Since the remainder is zero, $x - 1$ is a factor of $P(x)$.

b) Evaluate $P(-1)$.

$P(x) = x^3 - x^2 - 16x + 16$
$P(-1) = (-1)^3 - (-1)^2 - 16(-1) + 16$
$P(-1) = -1 - 1 + 16 + 16$
$P(-1) = 30$

Since the remainder is not zero, $x + 1$ is not a factor of $P(x)$.

c) Evaluate $P(-4)$.
$P(x) = x^3 - x^2 - 16x + 16$
$P(-4) = (-4)^3 - (-4)^2 - 16(-4) + 16$
$P(-4) = -64 - 16 + 64 + 16$
$P(-4) = 0$
Since the remainder is zero, $x + 4$ is a factor of $P(x)$.

d) Evaluate $P(4)$.
$P(x) = x^3 - x^2 - 16x + 16$
$P(4) = 4^3 - 4^2 - 16(4) + 16$
$P(1) = 64 - 16 - 64 + 16$
$P(1) = 0$
Since the remainder is zero, $x - 4$ is a factor of $P(x)$.

Chapter 3 Review Page 153 Question 8

a) For $P(x) = x^3 - 4x^2 + x + 6$, the possible integral zeros of the polynomial are the factors of the constant term, –6: ±1, ±2, ±3, and ±6. Test these values to find a first factor: $P(-1) = 0$. Use synthetic division to find the other factors.

$$\begin{array}{r|rrrr} +1 & 1 & -4 & 1 & 6 \\ - & & 1 & -5 & 6 \\ \hline \times & 1 & -5 & 6 & 0 \end{array}$$

Then, the remaining factor $x^2 - 5x + 6$ can be factored as $(x - 2)(x - 3)$.
So, $P(x) = (x + 1)(x - 2)(x - 3)$.

b) $P(x) = -4x^3 - 4x^2 + 16x + 16$ can be factored by grouping.
$P(x) = -4x^3 - 4x^2 + 16x + 16$
$P(x) = -4x^2(x + 1) + 16(x + 1)$
$P(x) = (x + 1)(-4x^2 + 16)$
$P(x) = -4(x + 1)(x^2 - 4)$
$P(x) = -4(x + 1)(x - 2)(x + 2)$

c) For $P(x) = x^4 - 4x^3 - x^2 + 16x - 12$, the possible integral zeros of the polynomial are the factors of the constant term, –12: ±1, ±2, ±3, ±4, ±6, and ±12. Test these values to find a first factor: $P(1) = 0$. Use synthetic division to find the other factors.

$$\begin{array}{r|rrrrr} -1 & 1 & -4 & -1 & 16 & -12 \\ - & & -1 & 3 & 4 & -12 \\ \hline \times & 1 & -3 & -4 & 12 & 0 \end{array}$$

Then, the remaining factor $x^3 - 3x^2 - 4x + 12$ can be factored by grouping as $(x - 3)(x^2 - 4)$.
So, $P(x) = (x - 1)(x - 3)(x - 2)(x + 2)$.

d) For $P(x) = x^5 - 3x^4 - 5x^3 + 27x^2 - 32x + 12$, the possible integral zeros of the polynomial are the factors of the constant term, 12: ±1, ±2, ±3, ±4, ±6, and ±12. Test these values to find a first factor: $P(1) = 0$. Use synthetic division to find the other factors.

–1	1	–3	–5	27	–32	12
–		–1	2	7	–20	12
×	1	–2	–7	20	–12	0

Repeat the process with the remaining factor $x^4 - 2x^3 - 7x^2 + 20x - 12$. Test factors of the constant term, –12: $P(2) = 0$.

–2	1	–2	–7	20	–12
–		–2	0	14	–12
×	1	0	–7	6	0

Repeat the process with the remaining factor $x^3 - 7x + 6$. Test factors of the constant term, 6: $P(1) = 0$.

–1	1	0	–7	6
–		–1	–1	6
×	1	1	–6	0

Then, the remaining factor $x^2 + x - 6$ can be factored as $(x + 3)(x - 2)$.
So, $P(x) = (x - 1)^2(x - 2)^2(x + 3)$.

Chapter 3 Review Page 154 Question 9

a) For $V(x) = 2x^3 + 7x^2 + 2x - 3$, the possible integral zeros of the polynomial are the factors of the constant term, –3: ±1 and ±3. Test these values to find a first factor: $P(-1) = 0$. Use synthetic division to find the other factors.

+1	2	7	2	–3
–		2	5	–3
×	2	5	–3	0

Then, the remaining factor $2x^2 + 5x - 3$ can be factored as $(2x - 1)(x + 3)$.
So, $V(x) = (x + 1)(2x - 1)(x + 3)$.
The possible dimensions of the block, in metres, are $(x + 1)$ by $(2x - 1)$ by $(x + 3)$.

b) When $x = 1$, the possible dimensions are 2 m by 1 m by 4 m.

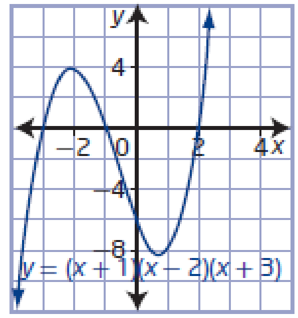
Chapter 3 Review Page 154 Question 10

Solve $P(-3) = 0$ to determine the value(s) of k.

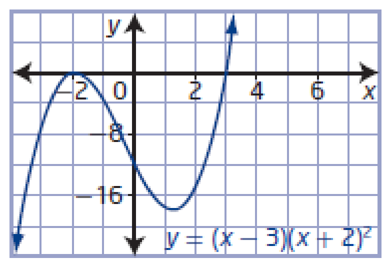
$$P(x) = x^3 + 4x^2 - 2kx + 3$$
$$0 = (-3)^3 + 4(-3)^2 - 2k(-3) + 3$$
$$0 = -27 + 36 + 6k + 3$$
$$-12 = 6k$$
$$k = -2$$

Chapter 3 Review Page 154 Question 11

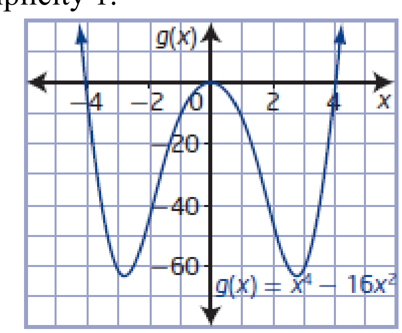
a) The function $y = (x + 1)(x - 2)(x + 3)$ is in factored form.
• So, the x-intercepts of the graph are –1, 2, and –3.
• The function is of degree 3 and the leading coefficient is positive. The graph of the function extends down into quadrant III and up into quadrant I.
• The zeros are –1, 2, and –3, each of multiplicity 1.
• The y-intercept is $(0 + 1)(0 - 2)(0 + 3)$, or –6.
• Use the end behavior and multiplicity of the zeros. The graph begins in quadrant III, passes through $x = -3$ to above the x-axis, down through $x = -1$ to under the x-axis, passes through the y-intercept and through $x = 2$ upward into quadrant I. The function is positive for values of x in the intervals $-3 < x < -1$ and $x > 2$. The function is negative for values of x in the intervals $x < -3$ and $-1 < x < 2$.



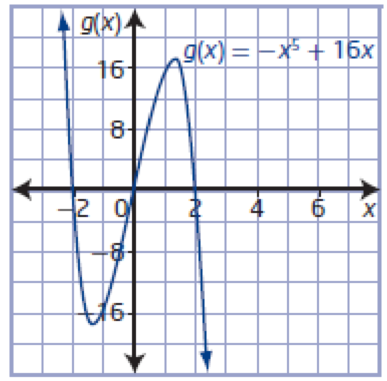
b) The function $y = (x - 3)(x + 2)^2$ is in factored form.
• So, the x-intercepts of the graph are –2 and 3.
• The function is of degree 3 and the leading coefficient is positive. The graph of the function extends down into quadrant III and up into quadrant I.
• The zeros are –2 (multiplicity 2) and 3 (multiplicity 1).
• The y-intercept is $(0 - 3)(0 + 2)^2$, or –12.
• Use the end behavior and multiplicity of the zeros. The graph begins in quadrant III, touches the x-axis at $x = -2$, goes down through the y-intercept and back up passing through $x = 3$ upward into quadrant I. The function is positive for values of x in the interval $x > 3$. The function is negative for values of x in the intervals $x < -2$ and $-2 < x < 3$.



c) Factor the polynomial expression $x^4 - 16x^2$.
$g(x) = x^4 - 16x^2$
$g(x) = x^2(x^2 - 16)$
$gx) = x^2(x - 4)(x + 4)$
• So, the x-intercepts of the graph are 0, 4, and –4.
• The function is of degree 4 and the leading coefficient is positive. The graph of the function extends up into quadrant II and up into quadrant I.
• The zeros are 0 (multiplicity 2) and 4 and –4, each of multiplicity 1.
• The y-intercept is a_0, or 0.
• Use the end behavior and multiplicity of the zeros. The graph begins in quadrant II, passes through $x = -4$ to below the x-axis, touches the x-axis at $x = 0$, passes through $x = 4$ upward into quadrant I. The function is positive for values of x in the intervals $x < -4$ and $x > 4$. The function is negative for values of x in the intervals $-4 < x < 0$ and $0 < x < 4$.



d) Factor the polynomial expression $-x^5 + 16x$.
$g(x) = -x^5 + 16x$
$g(x) = -x(x^4 - 16)$
$g(x) = -x(x^2 - 4)(x^2 + 4)$
$g(x) = -x(x - 2)(x + 2)(x^2 + 4)$
• So, the x-intercepts of the graph are 0, 2, and –2.
• The function is of degree 5 and the leading coefficient is negative. The graph of the function extends up into quadrant II and down into quadrant IV.
• The zeros are 0, 2, and –2, each of multiplicity 1.
• The y-intercept is a_0, or 0.
• Use the end behavior and multiplicity of the zeros. The graph begins in quadrant II, passes through $x = -2$ to below the x-axis, passes through $x = 0$ to above the x-axis, and passes through $x = 2$ downward into quadrant IV. The function is positive for values of x in the intervals $x < -2$ and $0 < x < 2$. The function is negative for values of x in the intervals $-2 < x < 0$ and $x > 2$.



Chapter 3 Review Page 154 Question 12

a) Compare the functions $y = 2(-4(x - 1))^3 + 3$ and $y = a(b(x - h))^n + k$ to determine the values of the parameters.

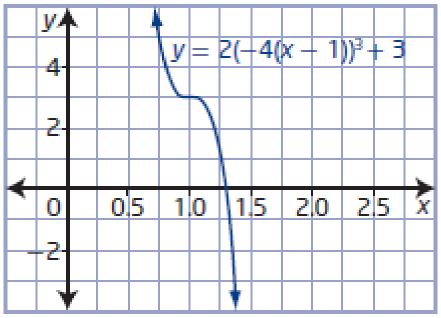
$b = -4$ corresponds to a horizontal stretch of factor $\frac{1}{4}$ and a reflection in the y-axis.

$a = 2$ corresponds to a vertical stretch of factor 2.
$h = 1$ corresponds to a translation of 1 unit to the right and $k = 3$ corresponds to a translation of 3 units up.

b)

Transformation	Parameter Value	Equation
horizontal stretch/ reflection in the y-axis	–4	$y = (-4x)^3$
vertical stretch/ reflection in the x-axis	2	$y = 2(-4x)^3$
translation left/right	1	$y = 2(-4(x - 1))^3$
Translation up/down	3	$y = 2(-4(x - 1))^3 + 3$

c)



Chapter 3 Review Page 154 Question 13

a) Since the graph extends down into quadrant III and up into quadrant I, the sign of the leading coefficient is positive. The x-intercepts are –3 (multiplicity 2) and –1. The function is positive for values of x in the interval $x > -1$. The function is negative for values of x in the intervals $x < -3$ and $-3 < x < -1$. The equation for the polynomial function is $y = (x + 3)^2(x + 1)$.

b) Since the graph extends down into quadrant III and down into quadrant IV, the sign of the leading coefficient is negative. The x-intercepts are –1 and 2 (multiplicity 3). The function is positive for values of x in the interval $-1 < x < 2$. The function is negative for values of x in the intervals $x < -1$ and $x > 2$. The equation for the polynomial function is $y = -(x + 1)(x - 2)^3$.

Chapter 3 Review Page 154 Question 14

a) For a quartic function with zeros –2, –1, and 3 (multiplicity 2), the corresponding factors are $(x + 2)$, $(x + 1)$, and $(x - 3)^2$.
Example: Two equations that satisfy this condition are: $y = (x + 2)(x + 1)(x - 3)^2$ and $y = -(x + 2)(x + 1)(x - 3)^2$.

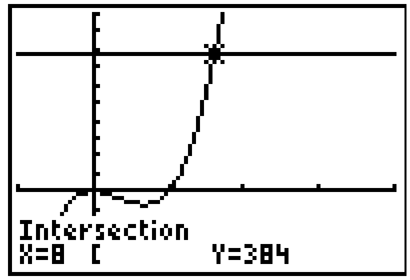
b) Determine the value of a so that the graph of $y = a(x + 2)(x + 1)(x - 3)^2$ passes through the point (2, 24).
$24 = a(2 + 2)(2 + 1)(2 - 3)^2$
$24 = a(4)(3)(1)$
$24 = 12a$
$a = 2$
So, the equation of the function that satisfies these conditions is
$y = 2(x + 2)(x + 1)(x - 3)^2$.

Chapter 3 Review Page 154 Question 15

a) Let x represent the length of the cardboard box. Then, the width is $x - 5$, the height is $2x$, and the equation for the volume is $V(x) = x(x - 5)(2x)$.

b) Solve $384 = x(x - 5)(2x)$ graphically.

The dimensions of the box are 8 cm by 3 cm by 16 cm.



Chapter 3 Practice Test

Chapter 3 Practice Test Page 155 Question 1

Odd-degree polynomials have at least one x-intercept. So, choice **C** is true.

Chapter 3 Practice Test Page 155 Question 2

Evaluate $P(1)$.
$P(x) = 3x^3 + 4x^2 + 2x - 9$
$P(1) = 3(1)^3 + 4(1)^2 + 2(1) - 9$
$P(1) = 3 + 4 + 2 - 9$
$P(1) = 0$
Since the remainder is zero, $x - 1$ is a factor of $P(x)$ and statement **B** is true.

Chapter 3 Practice Test Page 155 Question 3

For $P(x) = x^4 - 2x^3 - 7x^2 - 8x + 12$, the possible integral zeros of the polynomial are the factors of the constant term, 12: ±1, ±2, ±3, ±4, ±6, and ±12. Choice **D** is correct.

Chapter 3 Practice Test Page 155 Question 4

Consider A: Evaluate $P(1)$.
$P(x) = 2x^3 - 5x^2 - 9x + 18$
$P(1) = 2(1)^3 - 5(1)^2 - 9(1) + 18$
$P(1) = 2 - 5 - 9 + 18$
$P(1) = 6$
Since the remainder is not zero, $x - 1$ is not a factor of $P(x)$.

Consider B: Evaluate $P(-2)$.
$P(x) = 2x^3 - 5x^2 - 9x + 18$
$P(-2) = 2(-2)^3 - 5(-2)^2 - 9(-2) + 18$
$P(-2) = -16 - 20 + 18 + 18$
$P(-2) = 0$
Since the remainder is zero, $x + 2$ is a factor of $P(x)$.
Choice **B** is correct.

Chapter 3 Practice Test   Page 155   Question 5

For $y = 3\left(\frac{1}{4}(x-5)\right)^3 - 2$, $a = 3$, $b = \frac{1}{4}$, $h = 5$, and $k = –2$. To obtain this graph, the graph of $y = x^3$ will be vertically stretched by a factor of 3, horizontally stretched by a factor of 4, and translated 5 units to the right and 2 units down: choice **C**.

Chapter 3 Practice Test   Page 155   Question 6

a) $(x + 4)^2(x – 3) = 0$
$x + 4 = 0$ or $x – 3 = 0$
$x = –4$ $\quad x = 3$

b) $(x – 3)^3(x + 1)^2 = 0$
$x – 3 = 0$ or $x + 1 = 0$
$x = 3$ $\quad x = –1$

c) $(4x^2 – 16)(x^2 – 3x – 10) = 0$
$4(x – 2)(x + 2)(x – 5)(x + 2) = 0$
$x – 2 = 0$ or $x + 2 = 0$ or $x – 5 = 0$
$x = 2$ $\quad x = –2$ $\quad x = 5$

d) $(9x^2 – 81)(x^2 – 9) = 0$
$9(x – 3)(x + 3)(x – 3)(x + 3) = 0$
$x – 3 = 0$ or $x + 3 = 0$
$x = 3$ $\quad x = –3$

Chapter 3 Practice Test   Page 155 Question 7

a) For $P(x) = x^3 + 4x^2 + 5x + 2$, the possible integral zeros of the polynomial are the factors of the constant term, 2: ±1 and ±2. Test these values to find a first factor: $P(–1) = 0$. Use synthetic division to find the other factors.

+1	1	4	5	2
–		1	3	2
×	1	3	2	0

Then, the remaining factor $x^2 + 3x + 2$ can be factored as $(x + 2)(x + 1)$.
So, $P(x) = (x + 1)^2(x + 2)$.

b) For $P(x) = x^3 – 13x^2 + 12$, the possible integral zeros of the polynomial are the factors of the constant term, 12: ±1, ±2, ±3, ±4, ±6, and ±12. Test these values to find a first factor: $P(1) = 0$. Use synthetic division to find the other factors.

–1	1	–13	0	12
–		–1	12	12
×	1	–12	–12	0

Then, the remaining factor $x^2 – 12x – 12$ cannot be factored.
So, $P(x) = (x – 1)(x^2 – 12x – 12)$.

c) $P(x) = –x^3 + 6x^2 – 9x$
$P(x) = –x(x^2 – 6x – 9)$
$P(x) = –x(x – 3)^2$

d) For $P(x) = x^3 - 3x^2 + x + 5$, the possible integral zeros of the polynomial are the factors of the constant term, 5: ±1 and ±5. Test these values to find a first factor: $P(-1) = 0$. Use synthetic division to find the other factors.

$$\begin{array}{r|rrrr} +1 & 1 & -3 & 1 & 5 \\ - & & 1 & -4 & 5 \\ \hline \times & 1 & -4 & 5 & 0 \end{array}$$

Then, the remaining factor $x^2 - 4x + 5$ cannot be factored.
So, $P(x) = (x + 1)(x^2 - 4x + 5)$.

Chapter 3 Practice Test Page 156 Question 8

a) For $P(x) = x^4 + 3x^3 - 3x^2 - 7x + 6$, the possible integral zeros of the polynomial are the factors of the constant term, 6: ±1, ±2, ±3, and ±6. Test these values to find a first factor: $P(1) = 0$. Use synthetic division to find the other factors.

$$\begin{array}{r|rrrrr} -1 & 1 & 3 & -3 & -7 & 6 \\ - & & -1 & -4 & -1 & 6 \\ \hline \times & 1 & 4 & 1 & -6 & 0 \end{array}$$

Repeat the process with the remaining factor $x^3 + 4x^2 + x - 6$. Test factors of the constant term, –6: $P(1) = 0$.

$$\begin{array}{r|rrrr} -1 & 1 & 4 & 1 & -6 \\ - & & -1 & -5 & -6 \\ \hline \times & 1 & 5 & 6 & 0 \end{array}$$

Then, the remaining factor $x^2 + 5x + 6$ can be factored as $(x + 2)(x + 3)$.
So, $y = (x + 1)^2(x + 2)(x + 3)$.

Use a table to organize information about the function. Then, use the information to sketch the graph.

Degree	4
Leading Coefficient	1
End Behaviour	extends up into quadrant II and up into quadrant I
Zeros/*x*-Intercepts	−1 (multiplicity 2), –2, and –3
***y*-Intercept**	6
Intervals Where the Function Is Positive or Negative	positive values of y in the intervals $x < -3$, $-2 < x < -1$, and $x > -1$ negative values of y in the interval $-3 < x < -2$

This matches graph **B**.

b) $y = x^3 - 4x^2 + 4x$
$y = x(x^2 - 4x + 4)$
$y = x(x - 2)^2$

Use a table to organize information about the function. Then, use the information to sketch the graph.

Degree	3
Leading Coefficient	1
End Behaviour	extends down into quadrant III and up into quadrant I
Zeros/*x*-Intercepts	0 and 2 (multiplicity)
***y*-Intercept**	0
Intervals Where the Function Is Positive or Negative	positive values of $f(x)$ in the intervals $0 < x < 2$ and $x > 2$ negative values of $f(x)$ in the interval $x < 0$

This matches graph **C**.

c) $y = -2x^3 + 6x^2 + 2x - 6$
$y = -2x^2(x - 3) + 2(x - 3)$
$y = -2(x - 3)(x^2 - 1)$
$y = -2(x - 3)(x - 1)(x + 1)$

Use a table to organize information about the function. Then, use the information to sketch the graph.

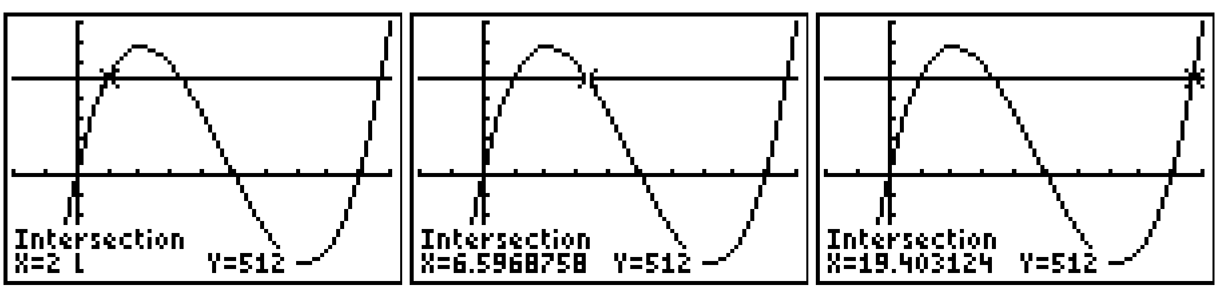
Degree	3
Leading Coefficient	–2
End Behaviour	extends up into quadrant II and down into quadrant IV
Zeros/*x*-Intercepts	–1, 1, and 3
***y*-Intercept**	–6
Intervals Where the Function Is Positive or Negative	positive values of $f(x)$ in the intervals $x < -1$ and $1 < x < 3$ negative values of $f(x)$ in the intervals $-1 < x < 1$ and $x > 3$

This matches graph **A**.

Chapter 3 Practice Test Page 156 Question 9

a) The volume of the box is given by $V(x) = x(20 - 2x)(18 - x)$.

b) Solve $512 = x(20 - 2x)(18 - x)$ graphically.



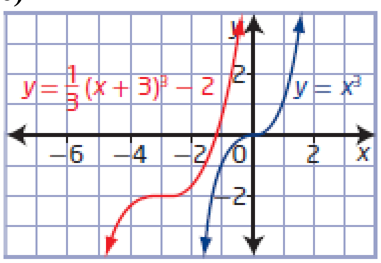
The possible whole-number dimensions of the box are 2 cm by 16 cm by 16 cm.

Chapter 3 Practice Test Page 156 Question 10

a) Compare the functions $y = \frac{1}{3}(x + 3)^3 - 2$ and $y = a(b(x - h))^n + k$ to determine the values of the parameters.
$b = 1$ corresponds to no horizontal stretch.
$a = \frac{1}{3}$ corresponds to a vertical stretch of factor $\frac{1}{3}$.
$h = -3$ corresponds to a translation of 3 units to the left and $k = -2$ corresponds to a translation of 2 units down.

b) The domain is $\{x \mid x \in \mathrm{R}\}$ and the range is $\{y \mid y \in \mathrm{R}\}$.

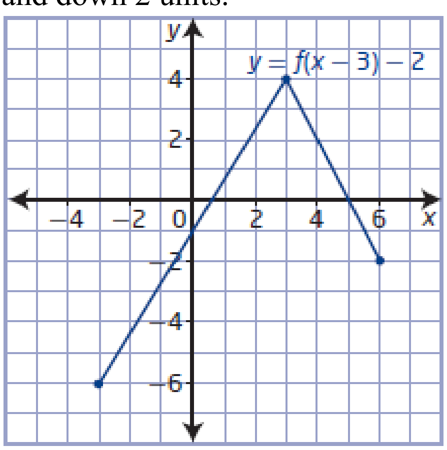
c)



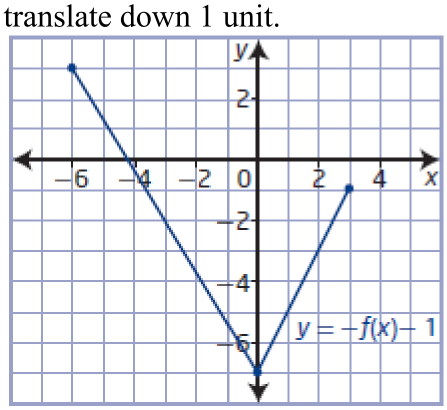
Cumulative Review, Chapters 1–3

Cumulative Review Page 158 Question 1

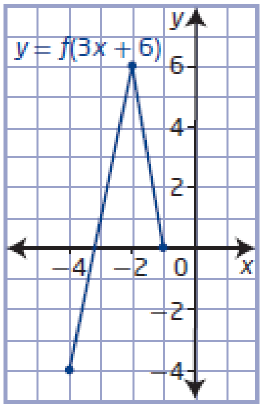
a) For the graph of $y + 2 = f(x - 3)$, translate the graph of $y = f(x)$ right 3 units and down 2 units.



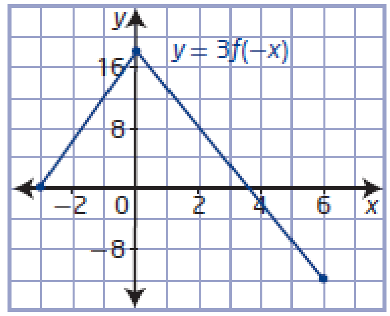
b) For the graph of $y + 1 = -f(x)$, reflect the graph of $y = f(x)$ in the x-axis and translate down 1 unit.



c) Rewrite $y = f(3x + 6)$ as $y = f(3(x + 2))$. For the graph of $y = f(3(x + 2))$, stretch the graph of $y = f(x)$ horizontally by a factor of $\frac{1}{3}$ and translate left 2 units.



b) For the graph of $y = 3f(-x)$, stretch the graph of $y = f(x)$ vertically by a factor of 3 and reflect in the y-axis.



Cumulative Review Page 158 Question 2

Locate key points on the graph of $f(x)$ and their image points on the graph of $g(x)$.
$(-2, 4) \rightarrow (1, 0)$
$(0, 0) \rightarrow (3, -4)$
$(2, 2) \rightarrow (5, 0)$
The orientation is unchanged, the graph has not been reflected.
The overall width or height have not changed, so the graph has not been stretched. Since the point (0, 0) is not affected by stretches, the graph has been translated 3 units to the right and 4 units down.
So, $a = 1$, $b = 1$, $h = 3$, $k = -4$, and the equation of the transformed graph is $y + 4 = f(x - 3)$.

Cumulative Review Page 158 Question 3

a) For $g(x) = f(x + 1) - 5$, $a = 1$, $b = 1$, $h = -1$, and $k = -5$. The graph of $y = f(x)$ is translated 1 unit to the left and 5 units down.

b) For $g(x) = -3(x - 2)^2$, $a = -3$, $b = 1$, $h = 2$, and $k = 0$. The graph of $f(x) = x^2$ is vertically stretched about the x-axis by a factor of 3, reflected in the x-axis, and then translated 2 units to the right.

c) Rewrite $g(x) = |-x + 1| + 3$ as $g(x) = |-(x - 1)| + 3$.
For $g(x) = |-(x - 1)| + 3$, $a = 1$, $b = -1$, $h = 1$, and $k = 3$. The graph of $f(x) = |x|$ is reflected in the y-axis and then translated 1 unit to the right and 3 units up.

Cumulative Review Page 158 Question 4

a) For $h(x) = f(x - 3) + 1$, $a = 1$, $b = 1$, $h = 3$, and $k = 1$. Use the mapping $(x, y) \rightarrow (x + 3, y + 1)$. So, the key point (6, 9) becomes the image point (9, 10).

b) For $i(x) = -2f(x)$, $a = -2$, $b = 1$, $h = 0$, and $k = 0$. Use the mapping $(x, y) \rightarrow (x, -2y)$. So, the key point (6, 9) becomes the image point (6, –18).

c) For $j(x) = f(-3x)$, $a = 1$, $b = -3$, $h = 0$, and $k = 0$. Use the mapping $(x, y) \rightarrow \left(-\frac{1}{3}x, y\right)$.

So, the key point (6, 9) becomes the image point (–2, 9).

Cumulative Review Page 158 Question 5

a) For $y = f(3x)$, $a = 1$, $b = 3$, $h = 0$, and $k = 0$. Use the mapping $(x, y) \rightarrow \left(\frac{1}{3}x, y\right)$.

Key Point	Image Point
(–4, 0)	$\left(-\frac{4}{3}, 0\right)$
(6, 0)	(2, 0)
(0, –3)	(0, –3)

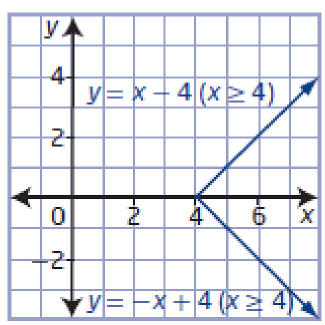
b) For $y = -2f(x)$, $a = -2$, $b = 1$, $h = 0$, and $k = 0$. Use the mapping $(x, y) \rightarrow (x, -2y)$.

Key Point	Image Point
(–4, 0)	(–4, 0)
(6, 0)	(6, 0)
(0, –3)	(0, 6)

Cumulative Review Page 158 Question 6

a) Since the graph of $y = |x| + 4$ passes the vertical line test, it is a function.

b) Reflect the graph of $y = |x| + 4$ in the line $y = x$.



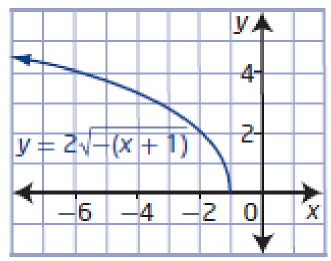
c) The graph of the inverse is not a function, since it fails the vertical line test. Example: A restricted domain for which the function has an inverse that is also a function is the right side of the V-shape: $\{x \mid x \geq 0, x \in \mathrm{R}\}$.
$f(x) = |x| + 4$, $x \geq 0$ and $f^{-1}(x) = x - 4$, $x \geq 4$

Cumulative Review Page 158 Question 7

Compare key points on the graph of $y = \sqrt{x}$ and their image points on the given graph.
$(0, 0) \rightarrow (-2, -3)$
$(4, 2) \rightarrow (0, -1)$
The overall width has changed, so the graph has been horizontally stretched by a factor of $\frac{1}{2}$. From the endpoint, the graph has been translated 2 units to the left and 3 units down.
So, $a = 1$, $b = 2$, $h = -2$, $k = -3$, and the equation of the transformed graph is
$y = \sqrt{2(x+2)} - 3$.

Cumulative Review Page 158 Question 8

For a vertical stretch by a factor of 2, horizontal reflection in the y-axis, and a horizontal translation of 1 unit to the left, $a = 2$, $b = -1$, $h = -1$, $k = 0$, and the equation of the transformed function is $y = 2\sqrt{-(x+1)}$.
domain $\{x \mid x \leq -1, x \in \mathrm{R}\}$
range $\{y \mid y \geq 0, y \in \mathrm{R}\}$



Cumulative Review Page 159 Question 9

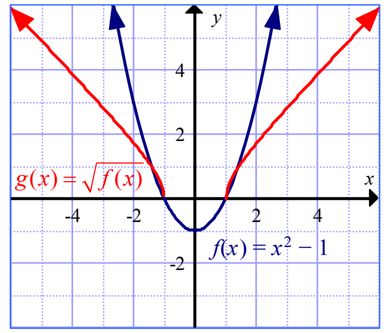
a) Compare key points on the graph of $f(x)$ and their image points on the given graph if $g(x)$ is a horizontal stretch of $f(x)$.
$(0, 0) \rightarrow (0, 0)$
$(9, 3) \rightarrow (1, 3)$
The overall width has changed, so the graph has been horizontally stretched by a factor of $\frac{1}{9}$. So, $a = 1$, $b = 9$, $h = 0$, $k = 0$, and the equation of the transformed graph is
$g(x) = \sqrt{9x}$.

b) Compare key points on the graph of $y = \sqrt{x}$ and their image points on the given graph if $g(x)$ is a horizontal stretch of $f(x)$.
$(0, 0) \rightarrow (0, 0)$
$(1, 1) \rightarrow (1, 3)$
The overall height has changed, so the graph has been vertically stretched by a factor of 3. So, $a = 3$, $b = 1$, $h = 0$, $k = 0$, and the equation of the transformed graph is $g(x) = 3\sqrt{x}$.

c) The equations from parts a) and b) are equivalent.
$g(x) = \sqrt{9x}$
$g(x) = \sqrt{9}\sqrt{x}$
$g(x) = 3\sqrt{x}$

Cumulative Review Page 159 Question 10

a) The x-intercepts of both graphs are –1 and 1. This is because points where $f(x) = 0$ are invariant.



b) For $f(x) = x^2 - 1$, the domain is $\{x \mid x \in \mathrm{R}\}$ and the range is $\{y \mid y \geq -1, y \in \mathrm{R}\}$. For $g(x) = \sqrt{f(x)}$, the domain is $\{x \mid x \leq -1 \text{ or } x \geq 1, x \in \mathrm{R}\}$, and the range is $\{y \mid y \geq -1, y \in \mathrm{R}\}$. The domain is restricted for $g(x)$ because it is a radical function.

Cumulative Review Page 159 Question 11

a) For $2x = \sqrt{x+3} - 5$, the restrictions on the variable are $x \geq -3$.

$$2x = \sqrt{x+3} - 5$$
$$2x + 5 = \sqrt{x+3}$$
$$4x^2 + 20x + 25 = x + 3$$
$$4x^2 + 19x + 22 = 0$$
$$(4x+11)(x+2) = 0$$

$4x + 11 = 0$ or $x + 2 = 0$

$4x = -11$ $\qquad$ $x = -2$

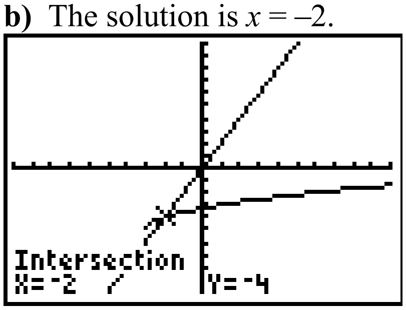
$x = -\dfrac{11}{4}$ or -2.75

Check. Substitute $x = -2.75$ and $x = -2$ into the original equation to identify any extraneous roots.

Left Side	Right Side	Left Side	Right Side
$2x$	$\sqrt{x+3} - 5$	$2x$	$\sqrt{x+3} - 5$
$= 2(-2.75)$	$= \sqrt{-2.75 + 3} - 5$	$= 2(-2)$	$= \sqrt{-2 + 3} - 5$
$= -5.5$	$= -4.5$	$= -4$	$= -4$
Left Side ≠ Right Side		Left Side = Right Side	

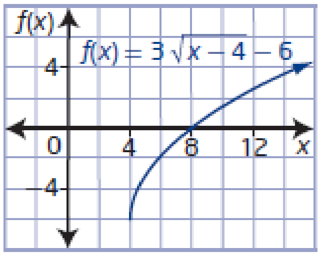
Ron is incorrect, $x = -2.75$ does not check. The solution is $x = -2$.

b) The solution is $x = -2$.



Cumulative Review Page 159 Question 12

a) The x-intercept is 8.



b)
$$0 = 3\sqrt{x-4} - 6$$
$$6 = 3\sqrt{x-4}$$
$$2 = \sqrt{x-4}$$
$$4 = x - 4$$
$$x = 8$$

c) The solutions or roots of a radical equation are equivalent to the x-intercepts of the graph of the corresponding radical function.

Cumulative Review Page 159 Question 13

a)

$$\begin{array}{r|l} & x^3 - x^2 + x + 2 \\ \hline x+1 & x^4 - 0x^3 - 0x^2 + 3x + 4 \\ & \underline{x^4 + x^3} \\ & \quad -x^3 - 0x^2 \\ & \quad \underline{-x^3 - x^2} \\ & \qquad x^2 + 3x \\ & \qquad \underline{x^2 + x} \\ & \qquad\quad 2x + 4 \\ & \qquad\quad \underline{2x + 2} \\ & \qquad\qquad 2 \end{array}$$

$$\frac{x^4 + 3x + 4}{x+1} = x^3 - x^2 + x + 2 + \left(\frac{2}{x+1}\right)$$

$P(x) = x^4 + 3x + 4$
$P(-1) = (-1)^4 + 3(-1) + 4$
$P(-1) = 1 - 3 + 4$
$P(-1) = 2$
The remainder when $x^4 + 3x + 4$ is divided by $x + 1$ is 2.

b)

$$
\begin{array}{r}
x^2+2x-5 \\
x+3\overline{)x^3+5x^2+x-9} \\
\underline{x^3+3x^2} \\
2x^2+x \\
\underline{2x^2+6x} \\
-5x-9 \\
\underline{-5x-15} \\
+6
\end{array}
$$

$$\frac{x^3+5x^2+x-9}{x+3}=x^2+2x-5+\frac{6}{x+3}$$

$P(x) = x^3 + 5x^2 + x - 9$
$P(-3) = (-3)^3 + 5(-3)^2 + (-3) - 9$
$= -27 + 45 - 3 - 9$
$= 6$
The remainder when $x^3 + 5x^2 + x - 9$ is divided by $x - 3$ is 6.

Cumulative Review Page 159 Question 14

For $P(x) = x^4 - 3x^3 - 3x^2 + 11x - 6$, the possible integral zeros of the polynomial are the factors of the constant term: –6: ±1, ±2, ±3, and ±6.

$P(x) = x^4 - 3x^3 - 3x^2 + 11x - 6$
$P(1) = 1^4 - 3(1)^3 - 3(1)^2 + 11(1) - 6$
$P(1) = 1 - 3 - 3 + 11 - 6$
$P(1) = 0$
The remainder when $x^4 - 3x^3 - 3x^2 + 11x - 6$ is divided by $x - 1$ is 0.

$P(x) = x^4 - 3x^3 - 3x^2 + 11x - 6$
$P(-1) = (-1)^4 - 3(-1)^3 - 3(-1)^2 + 11(-1) - 6$
$P(-1) = 1 + 3 - 3 - 11 - 6$
$P(-1) = -16$
The remainder when $x^4 - 3x^3 - 3x^2 + 11x - 6$ is divided by $x + 1$ is –16.

$P(x) = x^4 - 3x^3 - 3x^2 + 11x - 6$
$P(2) = 2^4 - 3(2)^3 - 3(2)^2 + 11(2) - 6$
$P(2) = 16 - 24 - 12 + 22 - 6$
$P(2) = -4$
The remainder when $x^4 - 3x^3 - 3x^2 + 11x - 6$ is divided by $x - 2$ is –4.

$P(x) = x^4 - 3x^3 - 3x^2 + 11x - 6$
$P(-2) = (-2)^4 - 3(-2)^3 - 3(-2)^2 + 11(-2) - 6$
$P(-2) = 16 + 24 - 12 - 22 - 6$
$P(-2) = 0$
The remainder when $x^4 - 3x^3 - 3x^2 + 11x - 6$ is divided by $x + 2$ is 0.

$P(x) = x^4 - 3x^3 - 3x^2 + 11x - 6$
$P(3) = 3^4 - 3(3)^3 - 3(3)^2 + 11(3) - 6$
$P(3) = 81 - 81 - 27 + 33 - 6$
$P(3) = 0$
The remainder when $x^4 - 3x^3 - 3x^2 + 11x - 6$ is divided by $x - 3$ is 0.

$P(x) = x^4 - 3x^3 - 3x^2 + 11x - 6$
$P(-3) = (-3)^4 - 3(-3)^3 - 3(-3)^2 + 11(-3) - 6$
$P(-3) = 81 + 81 - 27 - 33 - 6$
$P(-3) = 96$
The remainder when $x^4 - 3x^3 - 3x^2 + 11x - 6$ is divided by $x + 3$ is 96.

$P(x) = x^4 – 3x^3 − 3x^2 + 11x – 6$
$P(6) = 6^4 – 3(6)^3 – 3(6)^2 + 11(6) – 6$
$P(6) = 1296 – 648 – 108 + 66 – 6$
$P(6) = 600$
The remainder when $x^4 – 3x^3 − 3x^2 + 11x – 6$ is divided by $x – 6$ is 600.

$P(x) = x^4 – 3x^3 − 3x^2 + 11x – 6$
$P(−6) = (−6)^4 – 3(−6)^3 – 3(−6)^2 + 11(−6) – 6$
$P(−6) = 1296 + 648 – 108 – 66 – 6$
$P(−6) = 1764$
The remainder when $x^4 – 3x^3 − 3x^2 + 11x – 6$ is divided by $x + 6$ is 1764.

Cumulative Review Page 159 Question 15

a) For $P(x) = x^3 – 21x + 20$, the possible integral zeros of the polynomial are the factors of the constant term, 20: ±1, ±2, ±4, ±5, ±10, and ±20. Test these values to find a first factor: $P(1) = 0$. Use synthetic division to find the other factors.

–1	1	0	–21	20
–		–1	–1	20
×	1	1	–20	0

Then, the remaining factor $x^2 + x – 20$ can be factored as $(x – 4)(x + 5)$.
So, $P(x) = (x – 1)(x – 4)(x + 5)$.

b) For $P(x) = x^3 + 3x^2 – 10x – 24$, the possible integral zeros of the polynomial are the factors of the constant term, –24: ±1, ±2, ±3, ±4, ±6, ±8, ±12, and ±24. Test these values to find a first factor: $P(–2) = 0$. Use synthetic division to find the other factors.

+2	1	3	–10	–24
–		2	2	–24
×	1	1	–12	0

Then, the remaining factor $x^2 + x – 12$ can be factored as $(x + 4)(x – 3)$.
So, $P(x) = (x + 2)(x + 4)(x – 3)$.

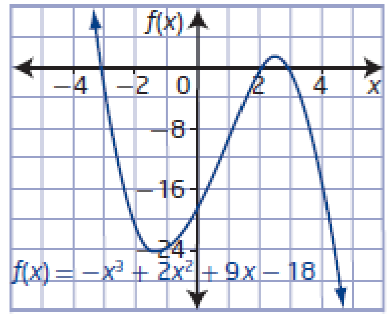
c) For $P(x) = –x^4 + 8x^2 − 16$, the possible integral zeros of the polynomial are the factors of the constant term, –16: ±1, ±2, ±4, ±8, and ±16. Test these values to find a first factor: $P(2) = 0$. Use synthetic division to find the other factors.

–2	–1	0	8	0	–16
–		2	4	–8	–16
×	–1	–2	4	8	0

The remaining factor $–x^3 – 2x^2 + 4x + 8$ can be factored.
$–x^3 – 2x^2 + 4x + 8$
$= –x^2(x + 2) + 4(x + 2)$
$= (x + 2)(–x^2 + 4)$
$= –(x + 2)(x^2 – 4)$
$= –(x + 2)^2(x – 2)$
So, $P(x) = –(x – 2)^2(x + 2)^2$.

Cumulative Review Page 159 Question 16

a) Factor $-x^3 + 2x^2 + 9x - 18$.



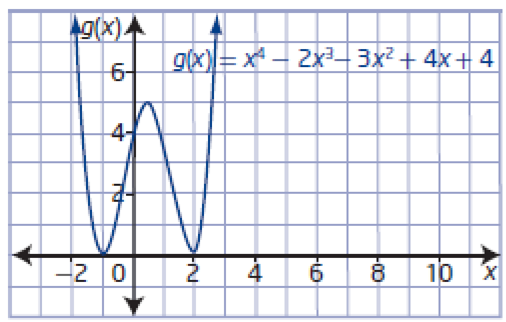
$-x^3 + 2x^2 + 9x - 18$
$= -x^2(x - 2) + 9(x - 2)$
$= (x - 2)(-x^2 + 9)$
$= -(x - 2)(x^2 - 9)$
$= -(x - 2)(x - 3)(x + 3)$
So, $f(x) = -(x - 2)(x - 3)(x + 3)$.
Then, the x-intercepts are 2, 3, and –3.
The y-intercept is –18.

b) For $g(x) = x^4 - 2x^3 - 3x^2 + 4x + 4$, the possible integral zeros of the polynomial are the factors of the constant term, 4: ±1, ±2, and ±4. Test these values to find a first factor: $P(-1) = 0$. Use synthetic division to find the other factors.

+1	1	–2	–3	4	4
–		1	–3	0	4
×	1	–3	0	4	0

Repeat the process with the remaining factor $x^3 - 3x^2 + 4$. Test factors of the constant term, 4: $P(2) = 0$.

–2	1	–3	0	4
–		–2	2	4
×	1	–1	–2	0



The remaining factor $x^2 - x - 2$ can be factored as $(x + 1)(x - 2)$.
So, $g(x) = (x + 1)^2(x - 2)^2$.
Then, the x-intercepts are –1 and 2.
The y-intercept is 4.

Cumulative Review Page 159 Question 17

a) For $V(x) = x^3 + 2x^2 - 11x - 12$, the given factor is $x + 1$. Use synthetic division to find the other factors.

+1	1	2	–11	–12
–		1	1	–12
×	1	1	–12	0

Then, the remaining factor $x^2 + x - 12$ can be factored as $(x + 4)(x - 3)$.
So, $V(x) = (x + 1)(x + 4)(x - 3)$.
The polynomials that represent the possible length and width of the box are $x + 4$ and $x - 3$.

b) When the height of the box is 4.5 m, $x = 3.5$. So, the possible dimensions are 4.5 m by 7.5 m by 0.5 m.

Cumulative Review Page 159 Question 18

If $f(x) = x^3$ is stretched vertically about the x-axis by a factor of 3, then reflected in the y-axis, and then translated horizontally 5 units to the right: $a = 3$, $b = -1$, $h = 5$, $k = 0$, and the equation of the transformed function is $g(x) = 3(-(x - 5))^3$.

Unit 1 Test

Unit 1 Test Page 160 Question 1

a) Locate key points on the graph of $f(x)$ and their image points on the graph of $g(x)$.
$(-4, 0) \rightarrow (-5, 1)$
$(-2, 4) \rightarrow (-4, 5)$
$(2, 3) \rightarrow (-2, 4)$
$(4, -2) \rightarrow (-1, -1)$

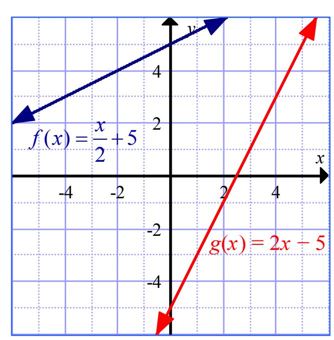
Since the orientation is unchanged, the graph has not been reflected in either axis.
The overall width has changed, so the graph has been horizontally stretched by a factor of 0.5. Since the x-intercept has changed, the graph has been translated 3 units to the left and 1 unit up. So, $a = 1$, $b = 2$, $h = -3$, $k = 1$, and the equation of the transformed graph is $g(x) = f(2(x + 3)) + 1$.
Choice **D** is correct.

Unit 1 Test Page 160 Question 2

For a reflection in the y-axis and a horizontal stretch about the y-axis by a factor of 3, $a = 1$, $b = -\frac{1}{3}$, $h = 0$, and $k = 0$. Use the mapping $(x, y) \rightarrow (-3x, y)$. The graph of the transformed function will have a different domain and different x-intercepts.
Choice **C** is correct.

Unit 1 Test Page 160 Question 3

Graph each pair of functions to check whether the graphs are reflections of each other in the line $y = x$.



The graphs of $f(x) = \frac{x}{2} + 5$ and $g(x) = 2x - 5$ are not inverses of each other.
Choice **D** is correct.

Unit 1 Test Page 160 Question 4

The function $y = |x + 4| - 3$ has domain $\{x \mid x \in \mathrm{R}\}$ and range $\{y \mid y \geq -3, y \in \mathrm{R}\}$.
Choice **A** is correct.

Unit 1 Test Page 160 Question 5

If the graph of $y = \sqrt{x+3}$ is reflected in the line $y = x$, then the range of this graph becomes the domain of the new graph, $\{x \mid x \geq 0, x \in \mathrm{R}\}$.
Choice **D** is correct.

Unit 1 Test Page 160 Question 6

If the graph of a polynomial function of degree 3 passes through (2, 4) and has x-intercepts of –2 and 3 only, then the factors are $(x + 2)$ and $(x - 3)$. The equation of the function is of the form $f(x) = a(x + 2)(x - 3)^2$, since $f(x)$ must be positive in the interval $-2 < x < 3$.
Determine the value of a so that the graph passes through the point (2, 4).
$4 = a(2 + 2)(2 - 3)^2$
$4 = a(4)(1)$
$4 = 4a$
$a = 1$
So, the equation of the function that satisfies these conditions is $f(x) = (x + 2)(x - 3)^2$ or $f(x) = x^3 - 4x^2 - 3x + 18$:.
Choice **C** is correct.

Unit 1 Test Page 160 Question 7

Cosider A: Evaluate $P(-1)$.
$P(x) = -x^3 - 4x^2 + x + 4$
$P(-1) = -(-1)^3 - 4(-1)^2 + (-1) + 4$
$P(-1) = 1 - 4 - 1 + 4$
$P(-1) = 0$
Since the remainder is zero, $x + 1$ is a factor of $P(x)$.
Choice **A** is correct.

Unit 1 Test Page 161 Question 8

Let $P(x) = x^4 + k$. Solve $P(-2) = 3$ to determine the value of k.
$P(x) = x^4 + k$
$3 = (-2)^4 + k$
$3 = 16 + k$
$k = -13$
When $x^4 + k$ is divided by $x + 2$, the remainder is 3. The value of k is –13.

Unit 1 Test Page 161 Question 9

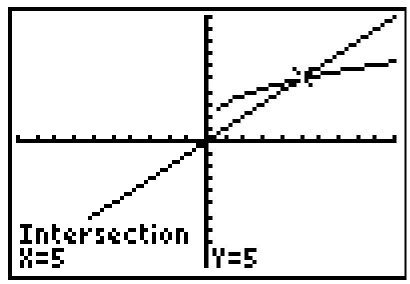
For the function $g(x) = f(x + 2) – 3$, only the value of the parameter k, –3, affects the range.
If the range of the function $y = f(x)$ is $\{y \mid y \geq 11, y \in \mathrm{R}\}$, then the range of the new function $g(x) = f(x + 2) – 3$ is $\boldsymbol{\{y \mid y \geq 8, y \in \mathrm{R}\}}$.

Unit 1 Test Page 161 Question 10

The mapping $(x, y) \rightarrow (x – 2, y + 3)$ represents a translation of 2 units to the left and 3 units up. So, $h = –3$ and $k = 3$.
The graph of the function $f(x) = |x|$ is transformed so that the point (x, y) becomes $(x – 2, y + 3)$. The equation of the transformed function is $g(x) = \boldsymbol{|x + 2| + 3}$.

Unit 1 Test Page 161 Question 11

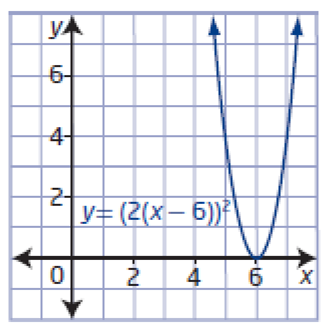
Solve $x = \sqrt{2x - 1} + 2$ graphically.



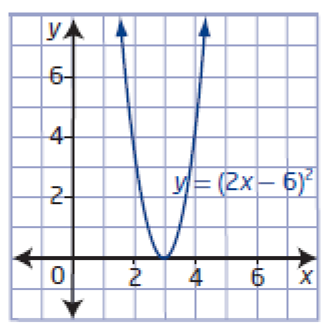
The root of the equation $x = \sqrt{2x - 1} + 2$ is $\boldsymbol{x = 5}$.

Unit 1 Test Page 161 Question 12

a) The graph of $y = x^2$ is stretched horizontally about the y-axis by a factor of $\frac{1}{2}$: $y = (2x)^2$.
Then, translated horizontally 6 units to the right: $y = (2(x – 6))^2$.



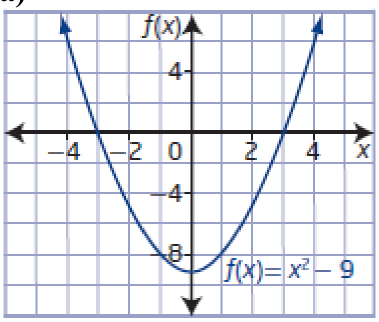
b) The graph of $y = x^2$ is translated horizontally 6 units to the right: $y = (x – 6)^2$.
Then, stretched horizontally about the y-axis by a factor of $\frac{1}{2}$: $y = (2x – 6)^2$.



c) From part b), rewrite $y = (2x - 6)^2$ as $y = (2(x - 3))^2$. The two graphs have the same shape but the vertex of the parabola in part a) is located at (6, 0) while the vertex of the parabola in part b) is located at (3, 0).

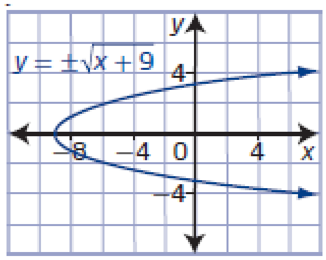
Unit 1 Test Page 161 Question 13

a)

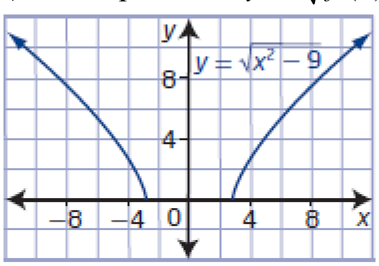


b) $f(x) = x^2 - 9$

$$y = x^2 - 9$$
$$x = y^2 - 9$$
$$x + 9 = y^2$$
$$y = \pm\sqrt{x+9}$$



c) The equation of $y = \sqrt{f(x)}$ is $y = \sqrt{x^2 - 9}$.



d) $f(x)$: domain $\{x \mid x \in \mathrm{R}\}$ and range $\{y \mid y \geq -9, y \in \mathrm{R}\}$
inverse of $f(x)$: domain $\{x \mid x \geq -9, x \in \mathrm{R}\}$ and range $\{y \mid y \in \mathrm{R}\}$
$\sqrt{f(x)}$: domain $\{x \mid x \leq -3 \text{ and } x \geq 3, x \in \mathrm{R}\}$ and range $\{y \mid y \geq 0, y \in \mathrm{R}\}$

Unit 1 Test Page 161 Question 14

For quadrant II: reflection in the y-axis, $y = f(-x)$.
For quadrant III: reflection in the y-axis and then the x-axis, $y = -f(-x)$.
For quadrant IV: reflection in the x-axis, $y = -f(x)$.

Unit 1 Test Page 161 Question 15

a) Mary should have subtracted 4 from both sides before squaring each side. She also incorrectly squared the right side in step 2.

$$2x = \sqrt{x+1} + 4$$
$$2x - 4 = \sqrt{x+1}$$
$$4x^2 - 16x + 16 = x + 1$$
$$4x^2 - 17x + 15 = 0$$
$$(4x-5)(x-3) = 0$$

$4x - 5 = 0$ or $x - 3 = 0$

$4x = 5$ $\quad x = 3$

$x = \frac{5}{4}$ or 1.25

Check. Substitute $x = 1.25$ and $x = 3$ into the original equation to identify any extraneous roots.

Left Side	Right Side	Left Side	Right Side
$2x$	$\sqrt{x+1}+4$	$2x$	$\sqrt{x+1}+4$
$= 2(1.25)$	$= \sqrt{1.25+1}+4$	$= 2(3)$	$= \sqrt{3+1}+4$
$= 2.5$	$= 5.5$	$= 6$	$= 6$

Left Side ≠ Right Side $\qquad$ Left Side = Right Side

The solution is $x = 3$.

b) Yes, John's method of graphing two functions representing each side of the equation can lead to a correct answer. He must find the x-coordinate of the point of intersection.

Unit 1 Test Page 161 Question 16

Solve $P(-3) = 0$ to determine the value(s) of k.

$P(x) = x^4 + 3x^3 + cx^2 - 7x + 6$

$0 = (-3)^4 + 3(-3)^3 + c(-3)^2 - 7(-3) + 6$

$0 = 81 - 81 + 9c + 21 + 6$

$-27 = 9c$

$c = -3$

For $P(x) = x^4 + 3x^3 - 3x^2 - 7x + 6$, $P(-3) = 0$. Use synthetic division to find the other factors.

$$\begin{array}{r|rrrrr} +3 & 1 & 3 & -3 & -7 & 6 \\ - & & 3 & 0 & -9 & 6 \\ \hline \times & 1 & 0 & -3 & 2 & 0 \end{array}$$

Repeat the process with the remaining factor $x^3 - 3x + 2$. Test factors of the constant term, 2: $P(1) = 0$.

–1	1	0	–3	2
–		–1	–1	2
×	1	1	–2	0

The remaining factor $x^2 + x - 2$ can be factored.
So, $P(x) = (x + 3)(x - 1)^2(x + 2)$.

Unit 1 Test Page 161 Question 17

a) For $P(x) = x^3 - 7x - 6$, the possible integral zeros of the polynomial are the factors of the constant term, –6: ±1, ±2, ±3, and ±6.

b) Test the values from part a) to find a first factor: $P(-1) = 0$. Use synthetic division to find the other factors.

+1	1	0	–7	–6
–		1	–1	–6
×	1	–1	–6	0

The remaining factor $x^2 - x - 6$ can be factored.
So, $P(x) = (x + 1)(x - 3)(x + 2)$.

c) The x-intercepts are –1, 3, and –2. The y-intercept is –6.

d) Since the leading coefficient is positive, the graph extends down into quadrant III and up into quadrant I. The function is greater than or equal to zero for values of x in the intervals $-2 \le x \le -1$ and $x \ge 3$.